ELEMENTS OF
DEDUCTIVE LOGIC

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ELEMENTS OF

DEDUCTIVE LOGIC

BY

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MENTS OF INDUCTIVE LOGIC" ETC.

Χρήσιμος ἡ πραγματεια πρὸς γυμνασιαν, πρὸς τὰς ἐντεύξεις
πρὸς τὰς κατὰ φιλοσοφιαν ἐπιστήμας.—ARISTOTLE

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PREFACE

THIS treatise is designed as a text-book for under-graduates. It comprises the body of approved logical doctrine, so that in a limited time a student may acquire a rounded knowledge of the fundamental forms of thought, be profited by the excellent discipline of the study, and prepared for the pursuit of philosophical sciences.

Those who wish to go beyond the elements of logic will find much additional matter in my larger work, entitled "The Theory of Thought," designed especially for universities. In my "Elements of Psychology" are explained the relation of the idea as a mental image to the notion as a product of thought, and the various mental processes involved in thinking. In both works many references will be found to authorities and to the literature of the subject.

In the preparation of the present text, I have tried to be clear, simple, and true, and to mitigate the natural severity of the subject by copious illustration. The care I have taken, and my experience

of more than twenty years in teaching logic, lead me to hope that my fellow-teachers and their pupils will find the treatise well adapted to their wants, and that it will therefore tend to promote the study of this admirable and invaluable science.

A special feature is a praxis appended to each chapter. Many standard exercises have been retained, and many new ones introduced. They have been carefully arranged in progressive order, in correspondence with the increasing complexity of the subject. I would suggest that the working of the praxes alone, without any recitation of the text, will insure a more satisfactory knowledge of elementary logic than the closest reproduction of the text, the praxes being omitted.

In the chapter on Fallacies, I have adhered to the original Aristotelic distribution, believing that it should be well known to every student of logic, and that none better has been proposed.

No treatment of induction is included in this book. But, deeply impressed with the importance of that branch of logic, especially in its relation to the physical sciences, I have prepared a companion volume, entitled Elements of Inductive Logic, which completes the system.

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CONTENTS

INTRODUCTION

I.—DEFINITION OF LOGIC

II.—PRIMARY LAWS

PART I.—CONCEPTION

I.—THE NOTION

PART II.—DEDUCTION

I.—IMMEDIATE INFERENCE

II.—THE SYLLOGISM

III.—CANON AND RULES

IV.—FIGURE AND MOOD

VIII.—QUANTITATIVE FORMS

IX.—FALLACIES

ELEMENTS OF
DEDUCTIVE LOGIC

INTRODUCTION

I.—DEFINITION OF LOGIC

§ 1. **Logic is the science of the necessary forms of thought.** The word logic is Greek. Aristotle, the author and finisher of the science, did not give this name to his work, but it was applied by his followers, and has been for many centuries its universally recognized title. In the mediæval universities, logic was studied as one of three ways to eloquence, and in modern schools it is justly held in high esteem as an independent science and an excellent discipline.

It will be well, at the outset, to have a distinct explication of the several terms used in the foregoing definition of logic, and to this we now proceed.

§ 2. A science is a complement of knowledge having, as to form, the character of logical perfection; as to matter, the character of truth. Logical perfection requires primarily that the objects

1

of knowledge shall be classified clearly, distinctly, completely, and harmoniously. Truth requires that the objects be real; what is unreal and false cannot constitute a science. Hence, a science is a perfected system of truths; or, science is classified knowledge. Few branches have reached this ideal perfection; perhaps pure mathematics alone has done so; but others, having made high attainments, are properly called sciences.

Science and art should be distinguished. <u>A science teaches us to know, an art to do</u>. Science discovers laws, art gives rules. Science is speculative, art practical. The scientist knows the proper relations of things, the artisan brings them into these relations. There is a science of civil law, there is an art for the practitioner. Anatomy is a science, surgery an art. But science often leads so directly to art, and art is so dependent on science, that they are not always clearly distinguishable.

Now, logic is not at all an art, but strictly a science. It tells us how we think when we think correctly, but does not pretend to tell us how to think. It is of great interest to know what are the principles and processes of thought, the laws that regulate intellect in the attainment of truth. Yet knowledge is power, and when one has mastered this science there is a practical result in a special cultivation of his faculties; for whatever process one clearly understands, it is manifest he can more efficiently perform. <u>As grammar and rhetoric are helpful to</u>

correct and elegant speaking and writing, so logic is helpful to correct and cogent thinking.

§ 3. The object-matter of logic is thought. Each science has its own object-matter. As astronomy treats of the stars, geology of the earth's crust, zoology of its fauna, botany of its flora, mathematics of quantity, theology of God, philosophy of principles, psychology of mind, ethics of morals, so logic treats of thought. Thought denotes the acts of the understanding as distinguished from perception, memory, imagination, feeling, desire, and volition, of whose exercises logic takes no notice. Thought is the bringing a notion into or under another. This is to comprehend or understand it. For example, when I say a lily is a flower, I bring my notion lily under a class-notion flower, and so this is a thought. Now, we think about all kinds of things, but logic is indifferent to all except one— that is, thought itself. In studying logic, we think about thought. As a science, it is the theory of thought.

Let it not be supposed, however, that logic treats of thought as exercised in scientific pursuits only. It treats of thought universally. Thought as found in all sorts of literature and speech, in common conversation, in silent meditation, all our every-day thinking about the most trivial things at any instant, as well as the lofty thought of the philosopher or theologian, is of the same nature, proceeds in the same manner, is according to the same laws, is

logical if correct.   Logic explains how any human
mind thinks correctly at any time about any thing.

§ 4. It appears, then, that logic has nothing to
do with the things we think about.   It treats of
thought in disregard of its content.   Excluding the
matter of thought, it discusses the form of thought.
The form as distinguished from the matter may be   •
exemplified thus : When I think that the book be-
fore me is a folio, the matter of this thought is
book and folio, the form is a judgment.   Thought
is concerned with the relations of objects to each
other, and the nomenclature of logic consists of the
names of these relations apart from the objects re-
lated ; as, judgment, concept and mark, species and
genus, subject and predicate, definition, syllogism,
dilemma, etc.   These are all names of mere forms
of thought.

In mediæval logic, the matter and form were dis-
tinguished as first and second intentions.   First in-
tentions are names of objects ; as, lily and flower,
book and folio.   Second intentions are names of
relations ; as, species and genus.   Hence a second
intention is, in modern logic, a form of thought.
Logic, then, is a science of second intentions.   Gram-
mar, also, is a science of second intentions, treating
of the forms of speech ; as, verb, adverb, noun, ad-
jective, clause, sentence, etc.   Grammar is the sci-
ence of second intentions or forms of speech.   Log-
ic is the science of second intentions or forms of
thought.

The matter and the form of thought cannot have any actually separate existence. No object is thinkable except under some form of thought; no form of thought can have any existence in consciousness unless there be some object of thought. But by abstraction we can contemplate these apart; we can consider either the object of thought or the manner of thinking it; we can distinguish the form from the content or matter. Logic, therefore, is an abstract science, abstracting from all matter the mere form of thought, and considering this only.

It follows that logic stands in a similar and fundamental relation to all other sciences, for it considers only what is common to all—that is, the forms of thought to which all are subjected—making that alone its object-matter. Now, philosophy is the science of principles, and therefore fundamental in treating of the primary truths that underlie all knowledge. But philosophy proceeds logically or not at all. Hence logic is fundamental even to philosophy, in that it exhibits the processes of thought which bind philosophy as well as all other sciences. Moreover, logic itself must proceed logically, and can become a science only by conforming to those laws which it is its province to explicate and exhibit.

§ 5. Logical forms are necessary forms. That is to say, the mind cannot think truly, unless it proceed according to these forms. It must not be understood that logic invents laws to control our

thinking; it merely discovers and unfolds the strict necessities that exist in the very nature of mind and things, and formulates them as laws of thought. It demonstrates that the mind must proceed according to these laws or under these forms, if the process be truly consecutive from one thought to another. Any violation of the laws, or deviation from the forms, it shows to be an inconsequence, and therefore futile.

For these laws are not necessary in the sense that they are inviolable. We may wilfully or ignorantly disregard them; and, blinded by prejudice or passion or confusion of thoughts, we often do violate them; but the process is fallacy and error, and the result null and void. All consequent thinking must be legitimate; that is, it necessarily conforms, consciously or unconsciously, to these laws. The conformity is necessary to valid thought. This is logical necessity (not unlike the practical necessity of a certain means to a certain end), and should be distinguished from philosophical necessity and moral necessity.

§ 6. Such, then, is the definition of Pure Logic, both Deductive and Inductive. Since it excludes the matter of thought, considering only its form, a strict observance of its limits would forbid the use of concrete examples. This would make the treatment very narrow, dry, and difficult. We shall therefore transgress the bounds of the definition whenever it seems desirable, and give concrete illus-

trations involving matter, hoping to enliven and facilitate the study. The student, however, should constantly keep in mind that logic has nothing to do with the matters thought about, does not at all concern itself with the truth or falsity of any propositions used for illustration, but deals only with the forms in which such matter is expressed.

II.—PRIMARY LAWS

§ 7. An analysis of our thoughts, discharging their matter, discovers that they have definite forms (§ 4). These forms, being native and necessary (§ 5), are universal; that is, they are in all thoughts, and all thoughts are in them. Since they are universal, we may view them as conforming to laws; and these, when formulated, are known as the laws of logic. Now, a thorough analysis of the empty forms, rejecting their differences, discloses certain general abstract principles. As the result of complete analysis, these are ultimate; as essential in every thought, even in that of themselves, they are necessary; as common to all the forms, they are strictly universal; as intuitively self-evident, they are axiomatic. These, then, are called Logical Principles, or Primary Laws of Thought.

This complement of laws is assumed by logic as its *punctum saliens*, and it proceeds to demonstrate from them as axioms the secondary and special laws that regulate all thinking. The whole of pure logic is only an articulate development of the primary laws and of their applications. Deductive logic posits three laws; inductive logic superadds others.

§ 8. The three primary laws are as follows:
The first is the LAW OF IDENTITY. It is the princi-
ple of affirmation. It is variously stated, but pref-
erably thus: **Whatever does not contradict a
subject may be affirmed of it.** The subject
and the attribute are thereby identified; hence the
name of this law. E. g., *A* is *A; 2×3=6; The
moon* is *our satellite; Francis Bacon* is *Lord Veru-
lam; Saltpetre* is *nitrate of potassa.* In these ex-
amples the identity in thought is entire.

But the law extends to partial identity. E. g.,
A is *a; 6 > 4; The moon* is *spherical; Congress* is
in session; Silver is *a metal.* In this case one term
is only a part of the content of the other. The
great majority of propositions take this form (§ 50).

Supplementary laws are: Whatever is essential
in a subject must be affirmed of it; as, *The sun* is
bright; and, Whatever is not essential in a subject
may be denied of it; as, *The sun* is not *up.*

Strictly logical propositions are always to be con-
strued literally, and should be distinguished from
rhetorical forms, wherein more is meant than meets
the ear. E. g., *A man's a man for a' that; What
I have written I have written; I am that I am.*
Such highly significant expressions in rhetorical
identity have no meaning when taken literally.

§ 9. The second is the LAW OF CONTRADICTION. It
is the principle of negation. Its statement is:
**Whatever contradicts a subject must be
denied of it.** Being in opposition, the subject

and an attribute are thereby set apart. <u>Contradic-</u>
<u>tories cannot coexist</u>; affirmations not self-consist-
ent are unintelligible. If we attempt to unite them,
the thought is null, it destroys itself. E. g., *A* is
not *A*, = *0 ; The circle is square ; The larger half ;
The laws of chance ; I expected to be disappointed ;
It is certain that nothing is certain.* This is the log-
ical paradox, or logical absurdity. Also notions
that are incongruous, as *noisy colors*, are essentially
contradictory, and cannot coexist.

According to the law, we must deny contradic-
tories of each other. Of two contradictories one
must be false. E. g., *A* is not *non-A ; 2 + 3 is not 4 ;
No pain is pleasurable ; What is wrong can never
be right ; No lie is of the truth.* Let it be observed
that *A* and *non-A* divide the universe of things, so
that whatever is one is not the other; everything
is either *man* or *non-man.* Such opposition is abso-
lute contradiction. But the members of a genus or
logical universe, though in themselves mere contra-
ries, are contradictory of each other relatively to
their limiting genus. Thus, if we take the universe
animal, then everything within this universe or
genus is either *man* or *non-man*, i. e. *brute*, and so
these are contradictories. E. g., *A man is not a
brute ;* likewise, *A fish is not a reptile ; A whale is
not a fish ; A vine is not a tree.* For similar rea-
sons, an attribute incongruous to a subject is to be
denied of it; as, *A dishonest man is not trust-
worthy.* Likewise two individuals are denied of
each other; as, *Francis Bacon is not Roger Bacon.*

Rhetorical contradictions are often used to convey emphatically a covert meaning. E. g., *Bitter Sweet ; Festina lente ; Not to decide* is *to decide ; When I am weak, then* am *I strong ; Hope that is seen* is not *hope ; In diplomacy, whatever is* is *something else ; Learned ignorance* is *wiser than presumptuous knowledge.* Such opposites are like the barbs of an arrow. The invisible point pierces, the barbs cling. This is the rhetorical paradox.

§ 10. The third is the LAW OF EXCLUDED MIDDLE. It prescribes a necessity in affirmation. A statement is: **Whatever contradicts a contradictory of a subject must be affirmed of it.** Evidently, of two absolute contradictories one must be true of any subject. If a genus or logical universe be strictly divided into two species, everything within it must be of one or the other kind. In either case no third affirmation is possible, i. e., every middle possibility is excluded; hence the name of this law. E. g., *X* is *either A or non-A ; God exists, or does not exist ; Every animal that* is not *a man* is *a brute ; Defence being impracticable, we must yield ; To be or not to be, that* is *the question ; If he do not fulfil the agreement, I shall be disappointed.* The argument called *reductio ad absurdum* (§ 108) is an application of this law. Of two contradictory alternatives it shows one to be absurd, hence the other must be allowed.

§ 11. The second and third laws are often united

§ 63.

in one brief but compound statement; as, <u>Of two contradictories one must be false, the other true</u>; or, Any attribute must be either denied or affirmed of any subject.

It has been proposed to reduce the three laws to one simple statement; as, All thought must be self-consistent. But an analysis of self-consistency will evolve the three laws as its ground. Still contradiction is obviously their common principle.

Also the attempt has been made to deduce from one the other two. But neither can be inferred as a second from another as first. In every such attempt the inferred law is necessarily presupposed,

§ 146.

which is *petitio principii*. Like the sides of a triangle, not only are they not the same, not reducible to unity, but also each gives, in its own existence, the existence of the other two. <u>The three are co-ordinate and complementary, distinct yet inseparable.</u>

§ 12. It has already been said that logic is concerned only with the form, not at all with the matter, of thought. Consequently, it furnishes no guarantee or criterion of the material truth of any proposition. There is no logical fault in our saying, for instance, that *Spain* is *an island*, or that *Theft* is *justifiable*. These false affirmations are in accord with the first law, and so are formally correct. What is conceivable in thought may be quite impossible in fact, and so is merely logically possible; as, *a centaur*. For the sphere of thought is

far wider than the sphere of reality, and there is no valid inference from the correctest thinking a thing to its actual existence.

But whatever violates either of these laws we know is impossible, not merely in thought, but in reality. We cannot allow that a thing can differ from itself, or that it can both be and not be, or that it can neither be nor not be. We must regard that as false and unreal which these laws condemn. They thus determine the sphere of impossibility, and that not merely in thought, but in reality; not only logically, but metaphysically.

While, then, these laws are no criterion of the reality of an object or of the truth of a proposition, they are a strict and universal criterion of non-reality and of falsity. Thus they are related to existence, not positively, but negatively. And this holds equally of all the secondary and special laws of logic. Our science, then, in its relation to other sciences, is not a positive criterion of truth; it is only a negative criterion, being conversant with thoughts, and not with things; with the possibility, and not with the reality, of existence.

§ 13. Beside the primary laws we place the POSTULATE OF LOGIC: **Logic postulates to state explicitly all that is implicit in a thought.** As pure logic has no concern at all with the matter of thought, so it has none with its language. It deals not in words, and must not be bound by them. Now, ordinary speech is often elliptical and rhetorical, much of thought being conveyed

in hints and metaphors. In dealing with it, the logician must be free to strip off all ornament, to supply all lacunæ, and so exhibit the thought naked and entire. This is sometimes difficult to do, thought being so subtile and evasive, and words so meagre and inaccurate. The only limitation is that the thought itself must not be changed. Also, there must be liberty to alter the form, provided, likewise, the thought be not modified. Expressions thus translated or transformed are equipollent, and the procedure is by equipollence.

§ 14. **Praxis.** What point or points of this chapter are obviously exemplified, and in what way illustrated, by each of the following propositions?

1. George Sand is a woman. He is she.
2. Courts of justice are worse than useless.
3. That which survives is the fittest.
4. When an irresistible force meets an insurmountable obstacle, the result is compound stationary motion.
5. Man is the only being that laughs.
6. Will is either free or necessitated.
7. That Herod is a fox, *means that* he is cunning.
8. Richard is himself again.
9. If death be death, these have passed into the past; If death be life, they live, though their semblance dies.
10. Summum jus, summa injuria.
11. A man who never makes mistakes, never makes anything else.
12. If a man be wise, he is cautious, *which is to say*, Every wise man is cautious.
13. His honor rooted in dishonor stood, And faith unfaithful kept him falsely true.

PART I.—CONCEPTION

I.—THE NOTION

§ 15. A notion is either a mark or a concept. In the forming of notions three movements of thought may be discerned: abstraction, generalization, and conception. First of abstraction.

When a complex object impresses us, it is apprehended as possessing qualities. In so far as they are dissimilar, they cause in us a feeling of difference. Now, if attention be fixed on one quality, as the color or the weight, the other qualities become obscure, while this one is drawn by attention into vivid consciousness, and so becomes the chief, perhaps the exclusive, object of cognition. This quality is said to have been abstracted, or drawn away from, the others, and the process is called logical abstraction. By it we obtain a clear and distinct knowledge of the qualities, attributes, characters, features, etc., that determine an object, or, in general, of its marks.

Marks considered merely in respect of their form are of several kinds, which may be designated and exemplified as follows:

1st. Positive and negative; as, *rational* is a positive, and *imperfect* a negative, mark of man.

2d. Essential or necessary, and accidental or contingent; as, *rational* is an essential, and *learned* an accidental, mark of man.

3d. Original and derivative; as, *rational* is an original, and *learned* a derivative, mark of man, derived from his rationality.

4th. Simple and complex; as, *conscious* is a simple mark, it being incapable of analysis, and *animal* a complex mark of man, this being composed of *organized* and *sentient*.

5th. Common and peculiar; as, *mortal* is a mark common to man and brute, *risible* a mark peculiar to man, found in no other being. A peculiar mark is called a property when viewed apart from the essence as belonging to a certain class of things, and to no other; as *risible* is a property of man, and a property of the circle is that *the chord of 60°* is *equal to the radius*. A peculiar mark is called a particular mark when it is found only in a single individual; as the mark set upon Cain.

A mark is very often thought of as though it were itself a substantial thing. Instead of being referred to its original substance, it is completely severed therefrom by thought, and established in an independent but fictitious existence. Marks so treated are called abstractions, and are expressed by abstract terms, very many ending in *-ness*. E. g., *blue* is a concrete mark of the sky, of the ocean, of sapphire, etc.; but *blueness* is thought of as something independent of these things and having a real existence apart, which is a mere fiction of

thought. Likewise, *Aristides* is *just*, but we extol *justice* apart from any person. Here the mark *just* is thought as concrete in the man, inhering in him; but *justice* is thought as abstract and having independent being. So *human* is a concrete, *humanity* an abstract term. A concrete term is the name of an inhering mark; an abstract term is the name of a mark viewed as an independent and substantial thing.

§ 16. In observing several objects, we note that they differ in some respects, or produce dissimilar impressions; perhaps we also note that they are alike in some respects, or produce similar impressions. The repetition of an impression is precisely what excites attention, and determines the direction of reflection. Thus consciousness is concentrated naturally on those objects which partially agree, and then on those respects or marks in which they agree. For example, we observe a horse, an ox, a goat, a dog, and we note that each has four feet, in which respect they agree. When marks are entirely similar the impressions they make on us are indistinguishable. But what we cannot distinguish is to us virtually the same. Accordingly, we consider them to be the same, though really in different objects. This act, to think the similar the same, is to generalize—is generalization. We think that each of the animals named above has the same mark, *four-footed*. A plurality is reduced to unity, and the generality of the mark consists

2

in this, that it may be said of any of the objects. Generalization is a fiction of thought, but without it our limited powers would be unable to grasp the multiplicity of objects about us.

Generalization is classification, another aspect of the same operation. By thinking a mark as common to several individuals, we thereby group them; we constitute a class. Thus, the animals named belong to the group or class *quadruped*.

Now, in considering this group of quadrupeds we note that the ox and goat each have horns; so we generalize and call them *horned quadrupeds*. The horse and dog have no horns; so we generalize and make a group of *non-horned quadrupeds*. This is specialization, correlative to generalization. We have marked off two species, the *horned* and the *non-horned*, the *A* and the *non-A*, subordinate to the genus or universe *quadruped*, which is their sum. It is obvious that specialization is the inverse of generalization, involves it, and likewise is classification.

§ 17. A third movement of thought is conception, its product a concept. To conceive is to grasp together. When a number of marks have been abstracted, they may be collected by thought into one notion, and so constitute a concept. A concept, then, is a union of marks, or a bundle of marks, thought as belonging to some thing.

Each object has an indefinite plurality of marks. Many of them may be known to us, but a mental

representation of an object becomes confused if we attempt to grasp into one or comprehend more than a very few of them. We therefore make a selection of some distinctive and some essential marks to form our concept, and must be content with this partial and inadequate representation. For example, I take the marks *Athenian, inquisitive, virtuous, moralist, famous, martyr*, these and perhaps others, to constitute my notion of *Socrates*. I may know much more about him, but practically this, or some such limited group of marks, comprises all I use in representing him. On the supposition that these marks have not been generalized, the concept is complex, but not general. Yet a notion thus formed of an individual is potentially general, potentially a class notion. There might be several persons having all the marks here attributed to Socrates. We must then add a particular mark, as, *Plato's teacher*, to the notion and thus secure its individuality.

When a concept is constituted of marks that have been generalized, that is, of common marks, the notion is then both complex and general. It is a class notion, comprising the objects to which the marks are common. For example, I take the following marks, which I have abstracted and generalized, each of which I have thought as common to a large number of objects: *self-luminous, bright, sparkling, celestial, very distant, relatively fixed*, etc. ; and, making a unity of this plurality, I form the concept *star*. This complex notion is applicable to

each of a host of distinct objects, in which fact its generality consists; and the word *star*, which stands for this bundle of marks, is the common name of many individual things. A general concept, then, is a combination or reduction to unity in thought of similar marks of objects, thereby constituting a class.

§ 18. The three momenta we have described are not separate and successive in thinking, but are so distinguished and stated to enable us to comprehend what is actually an indivisible operation. It is merely a logical analysis of an activity whose movements co-operate and coexist.

Moreover, a mark and concept are commutable. Every mark is potentially a concept, and every concept potentially a mark. Thus: *Man* is *animal,* or *Man* is *an animal.* Here *animal* is first a mark, then a concept. The distinction consists in the use made of the notion. If used connotatively, the notion is a mark; if used denotatively, the notion is a concept. *Man* is *animal* means that *man* has the attributes connoted by the mark *animal.* *Man* is *an animal* means that *man* is one of the kind of things denoted by the concept *animal.*

§ 19. A notion would immediately fall back into the infinitude and confusion from which it has been called out, were there not some especial means to render it permanent. This is accomplished by a word. The notion is fixed and ratified by a verbal

THE NOTION 21

sign, by means of which it can easily be recalled. Language, even in mere denomination, is a register of thought.

The name of a general notion is a common noun. Every common noun consists of one or more attributes belonging to each of several objects. It stands for a product of thought, and is a factitious unit useful in further thought. A mark is expressed by an adjective noun, a concept by a substantive noun, and an abstract noun is the name of a mark thought as a thing. Let it be observed that many notions, both marks and concepts, are registered in phrases instead of single words; as, for instance, there is no single word to express the notion of *morally weak*, or of *a rainy day*. Also, a verb is the naming of an action or passion or mere being.

A common noun is often used to designate an individual object or group by prefixing a limiting word; as, *a song, this world, those books, my house, the king, your friends, these troubles*, etc. Such naming designates the object, though individualized, as belonging to a class. The terms are connotative; they imply marks, and attribute these marks to the object or group they indicate.

A proper noun, strictly taken, is non-connotative. It denotes an individual, but in itself does not imply or indicate any qualities or marks of the individual. It is an unmeaning sign which we connect in our minds with an object, so that when it meets our eyes or ears it recalls to mind the thing. This is true of names strictly proper. But a name stand-

ing for a notion of an individual is evidently a complement of marks, as the example in § 17 of the notion *Socrates*. Moreover, names of individuals are often so contrived that they indicate their class; thus, names of persons generally distinguish sex, also family relations; and names of mere things also often have class significance, as *Monticello, Charlottesville, Fluvanna*. In such cases marks are connoted, and there is a distinct approach to the common noun or class name.

§ 20. Concepts have a twofold content, intensive and extensive. The intension is determined by the number of marks comprehended by the concept. E. g., *Man* connotes or comprehends the marks *existing, living, sentient, rational*. This explication of the connotation of a notion is its determination or definition. The quantity of extension is determined by the number of specific concepts or of objects contained under the concept. E. g., *Man* denotes or contains under it the species *logician, chemist, artist, mechanic*, etc. This explication of the denotation of a notion is its specification or division.

If the marks constituting the content of a concept be few, it may extend to many things; if the marks be many and distinctive, the concept extends to few things. Thus the concept *bird* has few marks, as *animal, biped, feathered, winged*, etc., but is applicable to, or contains under it, a great variety and number of things; now the concept *swan* has at least one more mark, *web-footed*, and

the variety and number of things denoted is less. Hence the LAW: **The greater the intension, the smaller the extension, and vice versa; or, these contents are in inverse ratio.**

We think a predicate either as a mark or as a class; as, *Facts* are *stubborn*, or, *Facts* are *stubborn things*. The one is thinking in intension, the other in extension. True, these involve each other, are essential correlatives, and are readily convertible; we do not think the one without, at the same time, thinking the other. But usually one mode is in vivid consciousness, while the other is obscure, and either phase of thinking may become habitual, one person more attentively considering the qualities of a thing, another regarding it as a member of a class.

§ 21. Progress in knowledge consists chiefly in rendering concepts clear and distinct. Conception is first obscure and then clear. We think a concept clearly when it is distinguished as a whole from other wholes. This is accomplished by negative judgments distinguishing or setting apart other concepts from this one, especially those which lie nearest to it, or by remarking a specific difference. E. g., We have a clear knowledge of the faces of our friends, since we readily know one from another. So we have a clear notion of *horse* when we know that it is not *ox*, nor *ass*, nor *mule*. So, also, our knowledge of *justice* is clear when we know that it is not *truth*, nor *benevolence*, nor *wisdom*, nor *power*.

Our notion of *perfume* is cleared by noting its specific difference; it is something that can be *smelled*.

Clear conception is first confused, then distinct. We think a concept distinctly when, viewing it as a plurality, we distinguish the marks or the objects that constitute it. Distinctness is attained by affirmative judgments. Analytic abstraction precedes, and is followed by a synthesis wherein the mark is affirmed of the thing. Or the notion is applied to its various objects, and in this becomes known by what is contained under it. E. g., An artist knows distinctly the features he has delineated. An artisan knows the virtues of his tools, and also their various kinds. It is natural and logical, when one undertakes to explain any obscure matter, to begin by clearing it, especially of those things that lie nearest to it—that is, which most nearly resemble it—showing that it is not these, and then proceeding to render it distinct by pointing out what it is in itself, or to what it applies.

Distinctness, then, has two modes: one which notes the marks which a notion connotes, distinctness in intension; the other which notes the objects it denotes, distinctness in extension. Intensive distinctness is attained by logical definition, which enumerates marks. Extensive distinctness is attained by logical division, which discovers kinds. A primitive notion, such as *identity*, can be cognized only *per se*. However clear it may be, it has no distinctness, either intensive or extensive.

§ 22. **Praxis.** Write answers to the following questions, and make reference to the section and paragraph illustrated:

1. Name the kinds of these marks of an apple: red (e. g., *positive, accidental, original, simple*), round, juicy with cider, innocuous, grown on this stem, worth five cents. Also of preachers as they ought to be, these: unselfish, called to this ministry, hortatory, devoted, well informed, spiritually minded, widely sympathetic, all things to all men.

2. Name which of the following terms are concrete and which abstract: truth, truthful, trueness, true, truthfulness, wisdom, wise, foolish, folly, consciousness, individuality, gratitude, homely, straight, a straight line, a circle, a fault, mercy, improved health, a healing balm.

3. What marks constitute your notion of Cæsar? What denotation has the word? What concept is formed of: small, hard, transparent, brilliant, elementary, precious, ornamental?

4. What mark is common to: chair, sofa, stool, bench? What general marks characterize the concepts: teacher, preacher, doctor, lawyer, author? What specific mark distinguishes teacher, preacher, and author from the others?

5. Change the quality noble into a concept. Distinguish the notion book from this book. Is Kaiser a common or proper name? Has the name Mary Jones Johnson any meaning?

6. Give the intension of the concepts: war-ship, hexagon, wisdom (see James iii. 17). Give the extension of the concepts: vessel, triangle, wisdom (cf. James iii. 15).

7. Clear the concept piano-forte; then render it distinct intensively, then extensively. Make a note on the logical procedure in 1 Cor. xiii.

II.—RELATIONS

§ 23. The relations which notions bear to each other need fuller explication. As preliminary, a very important and thorough-going distinction should be made between two wholes in or under which the mind thinks its objects. They are these:

1st. The Qualitative or Logical Whole. This is of two sorts:

(a) The intensive whole, whose parts are marks.

(b) The extensive whole, whose parts are kinds.

2d. The Quantitative or Mathematical Whole; of two sorts:

(a) The integral whole.

(b) The collective whole.

These primary forms of the notion, the qualitative and the quantitative, should be carefully observed. Heretofore we have considered solely the former (§ 15 sq.). It is entirely subjective, a creation of thought, and its parts are separable only by abstraction. It is general, and its parts are general. The latter is not so entirely subjective, since it is often determined by, and so corresponds to, an objective reality, and its parts are separable only by dissection. It is individual, and its parts are individual.

The importance of this distinction is seen in that, although both forms intermingle in our thoughts, reasoning in one of these wholes is regulated by principles differing from those regulating it in the other. Radical defects in the common logical theory, as well as many superfluities, are due to a neglect of the distinction. The oversight occurs, probably, because nearly every notion is capable of being viewed in either whole, either as a qualitative common notion or as a quantitative total; and its transference from one of these aspects or forms of thought to the other is often very facile, taking place almost unconsciously. This does not make it a matter of indifference, but is a reason why we should the more carefully note this subtile play of thought, so as not to be misled by it into illogical confusion.

We shall proceed to discuss the qualitative or logical whole minutely and at length. In the next section, however, and occasionally, we shall make mention of the quantitative whole so far as is needful to distinguish it clearly, and to recognize it when it occurs in qualitative propositions. Its full discussion is postponed to § 125 sq.

§ 24. The quantitative or mathematical whole, then, is individual; that is, not capable of division into kinds. An individual is *indivisum in se, et divisum ab omni alio.* Formally, it is a unit viewed as a quantity, and consisting of portions severable in thought. These are evolved by cutting asunder

the whole; that is, by partition or section, which must be clearly distinguished from logical division. Such parts are neither marks nor kinds, but merely new individuals.

First, the integral whole is that in which the whole is before the parts. The sections may be homogeneous, as a *hexagon* severed into *similar triangles;* or heterogeneous, as a *human body*, consisting of *head, trunk*, and *limbs*. Anatomy is a science of partition or dissection. The general notion *sword* logically divides into the kinds *sabre, rapier*, etc.; but each *sword* consists of and is separated by thought into the sections *hilt, blade*, etc.

Second, the collective whole is that in which the parts are before the whole. Such are the notions of an *army*, a *forest*, a *town*, formed by repetition of the notions of a *soldier*, a *tree*, a *house*. We should not confuse the general notion of *army*, which is a class notion capable of division into kinds, with the particular notion of some one *army*, which is an individual, and can only be parted into sections, as *regiments*. These are not kinds of *army*, but each is a new individual.

Quantitative notions occur frequently as the subject of qualitative propositions, but never as the predicate.

§ 25. In the qualitative, intensive whole, notions are related as congruent, incongruent, and conflictive. Congruent notions are such as may coexist in thought. All identical notions are congruent, as

achromatic and *colorless*. Also many that are not identical, as *learned* and *virtuous*, *beauty* and *riches*. Incongruent notions are such as cannot unite in the same object, as *a musical rose, a blue Monday*. Aristotle asks, *Is happiness praiseworthy?* There is no answer, for the question has no meaning. It is an incongruous jumble. Conflictive notions deny each other, as *virtue* and *vice*, *beauty* and *deformity*, *rich* and *poor*. They are in opposition.

Of congruent notions one involves or comprehends others when these are marks connoted by it. Thus the notion *Socrates* involves both *famous* and *Athenian*. These are co-ordinate, being both immediately comprehended. But *Athenian* further involves *Greek;* and *Greek, European;* and *European, human*. It is evident that these are not equally proximate and immediate in *Socrates*, and that they are in the relation of part to whole. They are *partes intra partes;* yet each permeates and informs the whole. So *chalk* is both *white* and *brittle*, and these marks coexist throughout.

In the qualitative, extensive whole, notions have the relations of coextension, subordination, co-ordination, and intersection. These may be figured thus:

Coextension	$(G.S)$	Globe Sphere
Subordination	$(A(M))$	Animal Man
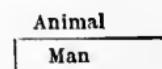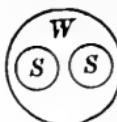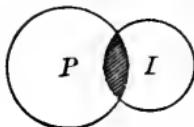
The circular notation needs no explanation. In the linear notation a horizontal line expresses the extension of a notion ; the comparative length and the relative position of two such lines, the relation of two notions. The vertical line indicates affirmation ; its absence, negation (§§ 86, 87).

§ 26. <u>Two notions are coextensive when they have the same denotation.</u> They may be symbolized by two coincident circles. The following are coextensive : *globe* and *sphere*, *triangle* and *trilateral*, *endogens* and *monocotyledons*, *double-refracting* and *polarizing crystals*, *to conquer one's passions* and *to become master of one's self*. Either of two such notions may be thought of as contained under the other. <u>Coextension should be distinguished from equality</u> which expresses quantitative relation.

§ 27. <u>One notion or concept is subordinate to or contained under another when it comprehends the same and more marks and extends to fewer objects</u> (§ 20). It is a species. Thus, *man* is a species of *animal*, and *sword* is a species of *weapon*. The former is subordinate to the latter; it connotes

more marks, but it denotes fewer objects. The superior concept, since more objects are contained under it, is the more general notion. It is a genus. Thus *animal* is the genus of *man ; weapon*, of *sword*. Both genera and species are classes, and the arrangement of things according to genera and species is classification (§ 16).

It is manifest that these forms of thought are merely relative. A genus may be contained under some higher concept, and then relatively to this higher genus it is a species. Thus *weapon* is a species of the genus *instrument*. A species may contain under it some lower concept, and then relatively to this lower species it is a genus. Thus *sword* is a genus of the species *sabre*. A notion that is thus alternately a genus relatively to lower, narrower concepts, and a species relatively to some higher, broader concept, is called a subalternate or subaltern genus. It is characterized as a genus that may become a species.

A genus is a universal notion or a universe (§ 9), since it turns the many parts into the unity of a whole. This is the logical meaning of universe, *ad unum versus*, giving *e pluribus unum*. It is often called, by way of eminence, a logical whole. A species is a special or specific notion, and since it is but a part of the generic whole it is a *particular* notion. The species as parts make up the genus as a whole (§ 10). These are *partes extra partes*, since they are distinct groups of objects; as *diamonds* and *rubies* are species of *jewels*. Hence we

can symbolize by circles or lines the relations of concepts in extension, but not of those in intension.

§ 28. **Praxis.** Write answers to the following questions, referring to the section and paragraph descriptive of the point in question:

1. In which whole do we think: the world, the planets, disorder, a flash, thunder, war, King Henry? Transfer each to the other whole.

2. What kind of quantity is: a constellation, a tree, a mob, Mt. Blanc, a sphere, a dollar, the ocean, this book?

3. What is the intensive relation of: money and memory, simple and complex, magnanimity and stature, an aching void, saint and sinner, sweet and sour, my dwelling-house is built of brick burned with fire?

4. What is the relation in extension of: brute and dog, heat and motion, seeing and perceiving, frankness and candor, lyric and hymn, hymn and sacred lyric, gun and cannon, bimana and mankind? Write the circular and linear notation in connection with each pair.

5. Considering chair, monarchy, and poetry, each as a subaltern genus, what genus is each contained under, and what species is contained under each?

6. What *partes extra partes* constitute the entire logical universe: animal, triangle, doctrine, lake, history, logical whole?

III.—DIVISION

§ 29. The relation of co-ordination is evolved by logical division (§ 25). It has already been seen that by specification we form subordinate groups whose members are co-ordinate. Since pure logic considers only the form, each genus or universal whole can contain only two species, marked with *A* and *non-A*. For *A* being a generic difference, that is, a mark not found in the genus or divisum, but found in some of its members, we know *a priori*, without any consideration of the matter of thought, that the members are exclusive of each other and exhaustive of the divisum. This is division by dichotomy, and the members are contradictories (§ 9). For example : *languages* are *Aryan* and *non-Aryan*, *animals* are *vertebrate* and *invertebrate*, the *ancients* were *Greeks* and *barbarians*. The process viewed intensively, as thinking marks in, is determination ; viewed extensively, as distinguishing species, it is specification (§ 20). In relation to each other, the two species are co-ordinate, being of equal rank in respect of the divisum ; but we remark that either may be of indefinitely greater extent or breadth than the other.

3

§ 30. The negative member of a dichotomy is characterized by the absence of the mark *A*, or, in other words, by the negative mark *non-A*. Hence arise negative, privative, or infinitated concepts. Often their sphere is very wide, denoting almost everything, and connoting very little, almost nothing positive. E. g., *unbounded, inert, apathy, blind, free, absolute, infinite*. In many cases a notion, originally a mere negative of its co-ordinate, has received a positive mark, so that either or both of the members of the dichotomy may be regarded as positive. E. g., *happy* and *unhappy, true* and *untrue* or *false, honor* and *dishonor, man* and *brute, town* and *country*, i. e., the *contrary*. Notions essentially negative, but whose name does not indicate this character, are often opposed by terms negative in form, yet positive in fact. Thus *temperate*, verbally positive, is a negative notion, opposed to the positive *intemperate*, which is negative in form. So also *ease* or *health* and *disease, pure* and *impure*.

Notions strictly correlative originate in dichotomy. The two always coexist in thought. We may be thinking more of one member of the couple than of the other, but if either exists the other coexists with it in consciousness, if either be expressed the other is implied. For example : *parent* and *child, ruler* and *subject, cause* and *effect, heavy* and *light, up* and *down, rich* and *poor, genus* and *species, positive* and *negative*. This last pair is the origin and generalization of all correlatives. One of the two is usually more or less negative, and

in case a separate name has not been adopted for each, the negative correlative to any positive notion may be expressed by the prefix *dis-*, *un-*, or *in-*, or the suffix *-ee* or *-less*. For example : *conscious* and *unconscious*, *correct* and *incorrect*, *truster* and *trustee*, *godly* and *godless*, *A* and *non-A*.

§ 31. In divisions not purely logical, but having respect to the matter, it often occurs that we have those that are more than dichotomous; we may have a trichotomy or a polytomy. E. g., *doctrines* are *helpful, harmless, hurtful*. This arises from two causes. Either it is an abbreviation, whereby several species, in turn subordinate, are condensed into one co-ordinate statement; as, *angles* are *right* (and *non-right*, which are) *acute* and *obtuse*. Or it arises from the lack of a sharp definition of our concepts. There is often between two opposite thoughts a notion or notions which it is impossible to identify surely with either, and so constituting a *tertium quid*, a third species, which it is needful to insert in order to exhaust the divisum. Thus we have *age* distributed as *young, middle-age, old ;* so also, *riches, competence, want ;* also, *white, gray, black.* For many of these intermediate species we have no name; as between *sick* and *well, strong* and *weak, long* and *short, wise* and *foolish.*

We have remarked that in a strictly logical distribution the members, *A* and *non-A*, are contradictories; no member of that universe can be both, or can be neither (§ 9 and § 29). In a trichotomy

or a polytomy the members are disparate notions.
Thus, *brook*, *creek*, *river* are disparates, contained
under the genus *streams*. Any two of such a di-
vision, as *brook* and *river*, are logical contraries; a
thing of this genus cannot be both, but may be
neither; it may be the *tertium quid*.

Finally, a polytomous division admits of one,
and only one, strictly privative or negative notion.
Thus, *some men lend, some borrow, some do both,
some do neither.* The intermediate ground, well
named the undefined or indifferent part, often
takes this negative character; as, *men* are *very in-
dustrious, positively lazy,* and *neither the one nor
the other.*

§ 32. A thoughtful consideration of the preced-
ing discussion, together with the illustrations, will
discover that each division is made with reference
to some general character or mark of the genus di-
vided. In dividing *animals*, for example, into *ra-
tional* and *irrational*, reference is made to their *in-
telligence*. Also in distributing *books* into *folios,
quartos*, etc., the reference is to their *size*. This
generic mark, or character of the divisum, which
reappears in a distinct, modified form as at once a
generic and a specific difference, is called the prin-
ciple or ground of the division, the *fundamentum
divisionis*.

A strict procedure, then, would be this: We as-
semble representative instances of the objects de-
noted by the divisum, and having fixed upon a gen-

eric mark as the principle of division, we select a mark immediately involving this principle to serve as a specific difference. Then we divide the denotation by affirming the specific difference of the class which it determines, and denying it of all other contained objects. In subsequent divisions we do likewise, involving in each new specific difference the one immediately preceding, and, of course, the original principle.

A nominal or artificial division is one made for some transient purpose, or to attain a practical end; or one tentative and precursory to a real division; or one popularly accepted and useful, such as the numbers that may be observed on every page, and in every few minutes of conversation. A real or scientific division is one proposing to divide notions and things according to their true and essential nature, in order to attain correct objective knowledge of things as they are. Such division develops natural kinds, and is to be looked for in the more refined sciences. The Linnæan artificial divisions of flora were precursory and tentative; those of Jussieu's natural system are real and more rigidly scientific.

§ 33. The old saying, *Divide et impera*, may be freely translated by Classify and conquer. In treating any matter, so great is the practical value of correct logical division, the root of classification, that we now gather up the foregoing principles in the following RULES:

1st. **The ground of a division should be an essential or at least an important mark of the divisum.** The ground or principle selected should be essential, if we would attain to real, scientific knowledge. It should be important, importing other attributes, if we would evolve an extended and valuable series. The purpose of an artificial division fixes its ground. In civil affairs it would be absurd to divide *men* into *horsemen* and *footmen*, but in military affairs this is important. In grammar, *words* are distributed according to syntactical relations; in a dictionary, alphabetically. Medical botany and the florist's manual distribute *plants* differently, and both differ from Jussieu. We sort our *books* by *size* to fit our shelves, by *subjects* for handy reference, by *binding* for show.

2d. **The members should, as parts, equal the whole divisum.** No one should exhaust the genus; as in *sciences* are *deductive* and *inductive*, whereas all sciences use deduction. Together they should exhaust it; which is the case in *angles* are *right* and *oblique*, but not in *governments* are *monarchies* and *democracies*, for there are other kinds.

3d. **The species should emerge immediately from the genus.** The genus should be proximate; as in *plants* are *flowering* and *flowerless*. Thought should not overlook and overleap subaltern genera, and proceed directly to remote species; as in *plants* are *annual*, *biennial*, and *perennial*. This rule relates chiefly to strict scientific

classification. In other matter the hiatus is quite usual and useful; as in *plants* are *noxious* and *innoxious*.

4th. **Only one principle should be used in determining a series.** The use of different grounds of division in a series gives rise to the logical fault called cross division. Thus: *vertebrates* are *quadrumana, bimana, quadrupeds*, and *bipeds*. Here two grounds of division are used, first *number of hands*, then *of feet*. We have, consequently, a cross division, *bimana* and *bipeds* are communicant species, they overlap in *man*.

Such a series is tested by dichotomy. Any correct trichotomy or polytomy may be reduced to a dichotomy by taking any one member as positive, and including the rest under its negative. Thus: *Substances* are *animal*, and *vegetable*, and *mineral*. Tested: S are a and *non-a* $(=v+m)$; or v and *non-v* $(=a+m)$; or m and *non-m* $(=a+v)$. This test applied to the following polytomy will demonstrate it to be logically vicious: *Religious sects* are *catholic, calvinist, episcopal*, and *dissenting*.

§ 34. **Praxis.** Write answers to the following questions and requisitions:

1. Which of these are positive, and which are negative notions, and what are the opposites of each : dry, simple, stranger, protestant, atheist, shadow, calm, disorder, sober, living, restless, iniquity, silence, unclean ?

2. What notions are correlate to : teacher, north, above, useful, right, committee, beggar, payer, pastor ?

3. What *tertium quid* lies between: day and night, hot and cold, love and hate, far and near, joy and sorrow? What is the logical relation of these notions?

4. What is the *fundamentum divisionis* of the following: conduct is interested and disinterested, animals are herbivorous and carnivorous, sounds are agreeable and disagreeable?

5. Assign a principle upon which: houses, fruits, historical periods, and tariff laws may each be dichotomized.

6. Criticise the following examples, that is, state whether they are logical divisions or quantitative partitions (§ 23). If divisions, state whether they are correct or not; and if not, what rule or rules are violated:

    (a.) The human hand consists of palm and fingers; it is flexible and expert; and known as right and left.
    (b.) Propositions are affirmative, hypothetical, and negative.
    (c.) Logic is deductive and inductive. The former treats of conception, deduction, and fallacy.
    (d.) Imaginative writers are poets, dramatists, and novelists.
    (e.) The seasons of the year are spring, summer, autumn, and winter.
    (f.) Men are rational and fanatic.
    (g.) Religions are Christian and Antichristian.
    (h.) Men are Americans, Europeans, blacks, and pagans.

7. Make several divisions of citizens, stating the ground of each, into the species: laity, aliens, peers, natives, clergy, commons.

8. Divide mankind on the principle of: age, sex, family relations, color, riches, education, occupation, and disposition.

IV.—DEFINITION

§ 35. The relation of intersection is discovered ℰ in definition (§ 25). Now, as division has reference primarily to extension, so definition refers primarily to intension. Our thought, having been cleared (§ 21), is by these rendered distinct; the external or extensive distinctness being secured by division, the internal or intensive distinctness by definition.

A definition is the explication of the essential and original marks of a concept, the definitum. Thus: *Man* is defined as *rational, sentient, living, existing.* It is evident, however, that this mode of statement is awkward, and in many cases impracticable. Observing, then, that the notion *animal* involves successively *sentient, living, existing,* we substitute for them that mark, and define summarily: *Man* is *rational* and *animal.* The mark *rational,* not included in the summation, is distinctive, since of all the notions that we here connote, it belongs to *man* alone. A logical definition, then, consists of two, and only two, essential and original marks, one being common, the other distinctive.

§ 36. From the foregoing principles are derived three corollaries, as follows:

1st. Simple notions, having no plurality of marks, are incapable of definition. The notion of *a being*, since it has only the one mark *existing*, and no differential or distinctive character, is indefinable, is an indefinite notion.

2d. An individual cannot be defined. Practically, we cannot enumerate its essential and original marks, or sum up all those it has in common with any other notion or thing. It can only be described.

3d. Since the definitum, or notion defined, contains implicitly the marks which its definition contains explicitly, they are reciprocating or convertible concepts. Thus: *A triangle is a polygon of three sides;* and, reciprocally, *A polygon of three sides is a triangle.* Hence either may replace the other. Thus: *Every rectilineal figure may be cut into triangles;* or, by replacement, *Every rectilineal figure may be cut into polygons of three sides.*

§ 37. Though definition relates primarily to intension, it is readily and usually viewed in relation to the extension of a concept. Concepts in extension often intersect; that is, two concepts often have a common part, and each a part not common (§ 25). Thus there are *Irish Protestants*, also there are *Irish* not *Protestants*, and *Protestants* not *Irish*. The common part is a species which is contained under either of the total concepts as a genus. In other words, whenever a certain group of things may be referred as a species to either of two

genera, these genera intersect, the group being a common part.

Now, the two portions of a definition may each be viewed as a concept in extension. If so, they will be seen to intersect, and the definitum to be the common part. Thus, the notion *rational being* intersects the notion *animal; man,* being both, is the common part. Formally, the definitum may be referred to either concept as a genus; logically, neither has preference; but whichever be chosen, the other serves to mark off or limit the definitum.
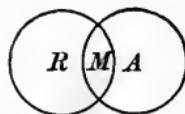
Thus we get the usual form of the logical definition. It consists of the genus proximate to the definitum, together with its specific difference. The proximate genus is that class under which the notion defined is immediately contained; as *animal* is the proximate genus to the concept *man.* The specific difference is that which thoroughly distinguishes the notion defined from all other species of that genus; as *rational* is the specific difference distinguishing *man* from all other species contained under *animal,* as *beasts, birds, fishes,* etc. Thus we have, *Man* is *a rational animal.* Also, *Logic* is *the science* (=prox. gen.) *of the necessary forms of thought* (=spc. dif.). Such is the formal definition *per genus et differentiam.*

§ 38. For the sake of clear treatment, it should at once be remarked that any predicate consisting

of two or more qualitative notions may be viewed as a genus with a difference. Thus: *Negroes* are *docile* (= dif.) *creatures* (= gen.). Here the genus is not proximate, and the difference is neither essential nor thorough-going. So, also: *Faith* is *the assurance of things hoped for*, etc. No clear thinker will mistake these for definitions.

A description recites constituent parts of a thing, especially such as are of interest and importance and at the same time distinctive, the selection being governed by a purpose. It may approximate definition.

An accidental mark or a property (§ 15) may be used to set a notion clearly apart; as, *Man* is *a featherless biped;* or, *Body* only is *mobile.* This is definite, but not definition.

A predicate generalizing the conditions, or the consequences, or explicating, not the connotation, but the denotation, is merely a quasi-definition. Thus: *Motion* is *the product of force and time; Malaria* is *feverous air; Mind* is *that which knows and feels, desires and wills.* This last is evidently a division rather than a definition. Such forms are often spoken of as definitions *a posteriori;* but a logical definition is strictly *a priori.*

§ 39. A real definition explicates the essence of a real object or class, in the forms of its proximate genus and specific difference. It is *a priori* and analytic. It has the character of a proposition affirming both the reality and the nature of the thing defined. Such are the verified defini-

tions of science; as, *Table-salt is sodium chloride; Attention is consciousness concentrated on an object.* Definitions of abstract notions derived from realities must also be accounted real; as, *A circle is a plane figure whose outline is everywhere equally distant from some point.*

A nominal definition is of the name of an object or class having only an ideal or hypothetical existence; as, *a centaur; open polar seas.* In practice the distinction between the nominal and the real cannot always be clearly applied; for nominal definitions, being often tentative and preliminary, may become real.

A genetic or causal definition is concerned with the rise or production of a thing, considering it, not as being, but as becoming. Thus: *A cone is a solid generated by the revolution of an angle about one of its sides.* The notion defined, not being given but made, this definition is *a priori* and synthetic.

§ 40. The original essence being known, help in making or criticising a definition is given by the following practical RULES:

A logically correct definition should be:

1st. **Positive.** The definition is always to be affirmed of the definitum. Negative statements serve to render the notion clear, and are important precursors to definition, but they do not render a notion distinct (§ 21). When the subject is a positive notion, which is most frequently the case, the

definition predicated of it should consist of positive notions. A definition should tell what a thing is, not what it lacks, or what it is not; as, *A line* is *length without breadth ;* and, *Pleasure* is *the feeling opposed to pain.* When, however, a notion is essentially negative, as *shadow, freedom, gentile, want,* then its definition should be negative; as, *Invertebrates* are *animals destitute of an internal skeleton.*

2d. **Adequate.** If the genus be not proximate, the definition is too wide; as, *Man* is *a rational being.* If the difference be not thorough-going— that is, not common to all members of the class— the definition is too narrow ; as, *Man* is *a praying animal.* A convenient test of adequacy is convertibility (§ 36).

3d. **Not tautological.** It should not contain the name of the thing defined, nor a synonym, nor a correlative term, for this is to define a thing by itself. Thus: *A lawmaker* is *one who makes law ; Life* is *the sum of the vital functions ; A cause* is *that which produces an effect.* Reciprocal definitions are not allowed ; as, *A board* is *a thin plank,* and *a plank* is *a thick board.* This is a sort of logical seesaw. It is called defining in a circle, and by the Greeks *diallelon* (through each other). There is a similar vice in reasoning called by the same name (§ 146).

4th. **Precise.** It should contain nothing merely accidental ; as, *The potato* is *the food of the Irish.* This difference is an accident (§ 15). It should contain nothing superfluous ; as, *A triangle* is *a figure*

having three sides and three angles. Here is superfluity. Names of forms should not be included with names of things; as, *The cur is a species of dog*, etc. Here *species* is superfluous. Derivatives being implied by their originals should be excluded as superfluous; as, *Honest dealing is rendering to every one his own property.* Here the notion of *own*, derived from *property*, is superfluous.

5th. **Perspicuous.** It should be intelligible, literal, and brief. A definition proposes to make a notion distinct; hence the use of notions more obscure than the one to be defined violates perspicuity; as, *The soul is the first entelechy of an organized body possessing life potentially.* Again, all figurative notions should be excluded, for tropes do not indicate what a thing is, but only something similar; as, *Omnipresence is a circle of which the centre is everywhere and the circumference nowhere.* Many expressions, however, originally metaphorical have become literal, and may properly be used in defining. Finally, brevity is certainly a merit, but extreme brevity may be less perspicuous than needless prolixity.

§ 41. **Praxis.** Analyze into genus and difference, classify by giving the kind, and criticise by applying the principles and rules, the following questionable definitions:

1. Philosophy is the science of principles.
2. Gratitude is the memory of the heart.
3. Motion is the change of place of body.

4. Motion is the act of potential being up to the measure of its potentiality.

5. Motion is an accidental property of body that effects a changing of its place.

6. Mad call I it; for, to define true madness,
What is't but to be nothing else but mad?

7. Green is a color compounded of blue and yellow.

8. Silence is the entire absence of sound or noise.

9. Mind is unextended substance.

10. Mind is conscious substance.

11. Health is the condition of a living body free from disease or pain.

12. A spheroid is a solid formed by the revolution of an ellipse about its diameter.

13. Opium is a vegetable product which causes sleep.

14. A dragon is a serpent breathing flame.

15. A synopsis is a conspectus of the chief points.

16. Animal is the genus denoting men and brutes.

17. Psychology is the science of the phenomena of mind.

18. Logic is the light-house of the understanding.

19. Dirt is matter in the wrong place.

20. Pleasure is the reflex of normal activity.

21. An atom is an ultimate particle of matter incapable of division.

22. A circle is a curved line returning upon itself, all the points of which are equidistant from a given point within called the centre.

23. A point is position without parts or magnitude.

24. Time is a measured portion of indefinite duration.

25. Laws are the expressed will of a ruler; and a ruler is one whose will is expressed in laws.

V.—SYSTEM

§ 42. As preliminary to an examination of logical system, we will present and remark upon the following scheme:

Intension.	Existing................ Minerals, Plants, Brutes, Men.	Extension.
	Existing, living................. Plants, Brutes, Men.	
	Existing, living, sentient ....Brutes, Men.	
	Existing, living, sentient, rational ....Men.	

The most obvious point here illustrated is the law that as intension increases, extension diminishes, and *vice versa ;* that the maximum of either is the minimum of the other; that the two are in inverse ratio (§ 20).

In ascending the series, we think marks out and think things in. This, on the intensive side, is abstraction (§ 15); on the extensive side, it is generalization or generification (§ 16).

In descending the series, we think marks in and think things out in the same mental act. This, on the intensive side, is determination; on the extensive side, it is specialization or specification.

§ 43. The same matter in a modified form, with some additions, is presented in the following scheme:

4

Second Intentions. Concepts of Forms.	First Intentions. Concepts of Things.	Intension or Depth. Marks connoted.	Extension or Breadth. Things denoted.
Summum Genus	Being or Thing	Existing	All Things
Species or Sub-genus	Organism	Ex., living	All Organisms
Species or Sub-genus	Animal	Ex., lv., sentient	All Animals
Infima Species	Man	Ex., lv., sn., rational	All Men
Individual	Aristotle	Ditto, Father of Logic	One Being

Here is represented a complete logical system founded on the relation of genus and species. It should be thoughtfully examined by the student of logic, in all its details, some of which we now proceed to discuss.

§ 44. It is evident that thought, rising from individuals to classes, and by successive generalizations forming wider and wider classes or genera, at each step diminishing the marks connoted, must at last reach a notion of widest generality, connoting but one mark and denoting all things, above which, of course, it cannot rise. This highest, widest notion is the *Summum Genus*, and is characterized as the genus that cannot become a species. It is represented in the foregoing scheme by *Being* or *Thing*, which are synonymous, comprehending only the mark *Existing*, and containing under it *All Things*.

It is possible to analyze metaphysically and logically the notion *being* or *thing* into its constituent notions *matter* and *form* (§ 4). It is therefore referable to the still higher genus *matter*, and so is definable thus: *A thing is matter having form*. In this view, *matter*, taken in its widest, metaphys-

ical sense, is the true *summum genus*. We shall, however, for convenience, continue to speak of *being* or *thing* as the actual *summum genus*, simple, indefinable, and ultimate.

In departments of science, it is not usual to make reference to this common genus. For each, its own subject is regarded as *summum genus*, that notion which is characterized by the mark selected as its *fundamentum divisionis* (§ 32). Thus, in botany, *plant* is the highest genus considered; in zoology, *animal;* in political economy, *wealth;* in logic, *thought form.* They leave to metaphysics or ontology, the science of being, the exploration of the still higher, more rarefied region. Similarly, in more commonplace matters, some subaltern genus (§ 27) is usually assumed as ultimate.

But the frequent use of the word *thing* shows what constant mental reference is had to the actual *summum genus.* Indeed, whenever we do not know the proximate or approximate genus of an object, or do not care to be exact, we mount up on eagle wing and call it a thing; thus: *A comet is a curious thing.* Likewise it is used as an index of mere existence; as, *Evil is an unavoidable thing.* Again, if we wish to consider an object relative to some one mark exclusively, we call it a thing; as, *Wine is a hurtful thing, because,* etc.; or if we wish to emphasize some mark; as, *Cruelty is a hateful thing.*

§ 45. On the other hand, when thought descends

the scale, and by successive specifications forming narrower and narrower classes or species, at each step adding marks and so rejecting things, it must at last reach a class of narrowest generality, connoting a maximum plurality of common marks, and denoting a minimum plurality of things, below which, of course, it cannot descend. This lowest, narrowest class is the *Infima Species*, and is characterized as the species that cannot become a genus. It is represented in the foregoing scheme by *Man*, comprehending many common marks, and containing under it only individual human beings.

The early logicians consider the *infima species* as fixed by nature, and expressed in the terms *man*, *horse*, etc. Such classes as *negro*, *barb*, etc., they do not admit to be species, but merely accidental varieties. But the whole question of natural kinds belongs exclusively to the naturalist, and with it the logician has nothing whatever to do. In logical theory, which disregards matter (§ 4), system is restricted only by the primary laws of thought. Hence division into logical kinds proceeds until no mark common to even two individuals remains to serve as a specific difference. The species that comprehends all the common marks is theoretically the *infima species*, for that alone cannot become a genus by further division.

§ 46. It is important to observe, clearly and distinctly, what relation individuals bear to the logical system. This system consists of classes only,

and hence an individual is not a member of it. For an individual is not a kind, is not a logical part (§ 24); that is, it cannot be evolved by division. It is evident *ex vi termini* that the *infima species* has no subordinate. It follows that the objects of which a class is formed cannot in strictness be spoken of as contained under the class, though this expression is used. More properly the class is said to denote, not only its species, but also the objects or things it comprises.

But individual objects are the basis of all classification (§§ 16, 32). Now, it is not necessary in generalizing that we should begin with the *infima species* and thence build up the scale. It is competent to begin with any wide class and evolve the system downward and upward. This indicates that any and every genus denotes, not only its subordinate classes, but also all the individual things of which it can be predicated.

An individual, as the word itself points out, is logically indivisible (§ 24). But this is a mark also of *infima species*. What, then, distinguishes the one from the other? Principally that the latter is a class, the former not. Thus, while the latter consists of common marks only, the former possesses also at least one particular mark, represented in the scheme by *Father of Logic*. This particular mark determines only a numerical, not a specific, difference; and therefore the individual cannot be defined, yet may be described (§ 38). Such is the logical individual (§ 17). The real, actual individ-

ual possesses also a distinct existence in space or
time. It is discriminated by perception, external
or internal. It has many numerical differences.

§ 47. It is apparent that division and definition
are correlative forms (§ 30). As division is con-
cerned with the extension or breadth of a concept,
so definition is concerned with its intension or
depth. By the one a notion is rendered externally
or extensively distinct; by the other it is rendered
internally or intensively distinct (§ 21). A division
explicates or evolves subordinate concepts; a defi-
nition explicates or evolves marks. The one devel-
ops the sphere, the other the comprehension. The
one analyzes the denotation, the other the conno-
tation. To each of the forms herein named is op-
posed a correlative.

In a systemized series of concepts, division looks
down, definition up, the scale. When a specific
subject is to be treated, we first define it; we give
its proximate genus, the one next above, which in-
volves all the marks of the preceding genera, in-
cluding the highest; we also give its specific differ-
ence, which sets it apart from co-ordinate notions.
Then we proceed downward, dividing and subdi-
viding, until we reach and include the lowest
species. This exhausts the scale, and the treat-
ment is logically complete. Of course it is not
necessary that this order should be rigidly ob-
served. In the progress of a treatise, definition
may often replace division, and one or the other

will preponderate according to the point in the scale at which a beginning is made, or according to the inclination of the thinker or the nature of his subject.

But since division and definition are convertible correlatives, a system may be expressed entirely in either, they being, *mutatis mutandis*, the same form. We may begin with the *summum genus*, and, descending, exhaust the scale by a series of divisions. Or we may begin with the *infima species*, and, ascending, exhaust the scale by a series of definitions. Any specific concept being defined, it is requisite to define the proximate genus to which it is referred, and again the proximate genus to which this is referred, and so on until the *summum genus* is reached.

Certain sciences, as botany and zoology, are sometimes called classificatory sciences, because they exhibit their matter mostly in the form of divisions. But all sciences are classificatory, and those referred to should rather be called dividing sciences. On the other hand, chemistry is eminently a defining science. Having named the elements, it uses few other names, a compound being designated generally by its definition only; as, *potassium iodide*, and *nitrate of cupric oxide*. It would be quite possible, however, to state the relations of chemical substances as genera and species.

Thus it is that thoughts are elaborated and rendered clear and distinct by being co-ordinated and subordinated, by being divided and defined, until

they are gradually built into systems, more or less complete and perfect. Let it be particularly remarked that this is true not merely of scientific thinking, but is equally true of our every-day thinking, and that about the most trivial matters (§ 3). It is thus that at all times and about all things we do think, and, governed by the necessary laws of pure thought, it is thus that we must think. Every common noun in a language occupies a place in some of the countless hierarchies of concepts which the human mind is forming or has formed. It is true that in most minds there is much confusion and disorder in the fabric of thought; still, the greater part of the humblest mental life is occupied in generalizing and specializing, in systematically arranging and correcting the arrangement of thoughts.

§ 48. A series of definitions may be expressed either directly, from the lowest species upward, or inversely, as in the examples given in the next section. A series of divisions is generally best expressed in the unnamed manner briefly exemplified in § 23, in the distribution of *Wholes*, which should, therefore, be closely considered. The early logicians made use of the figure of a logical tree, *arbor Porphyriana*, erect, horizontal, or inverted, from which comes the familiar phrase "branches of knowledge." Also they used the figure of a scale or ladder (*scala*, κλῖμαξ, a staircase). These several forms are illustrated in the next section.

§ 49. **Praxis.** The following exercises should be written with care, and reference made to the sections illustrated :

1. Name a number of individuals denoted by the wide notion *town*. Of these, which are comprised by the narrower notion *city?* Of which may *metropolis* be predicated? *Charlle. Rich? New York; R H. — N*

2. Criticise the following series : The U. S. domain consists of states and territories ; the states are Northern and Southern, the Northern are Eastern and Western, all are divided into counties, and these into townships. *Bad*

3. Divide and subdivide *officers of the U. S. government* with reference to their official functions. *Pres — VP. Cab*

4. What is the *fundamentum divisionis* of the following *scala?* Reduce it to a series of definitions, supplying specific differences:
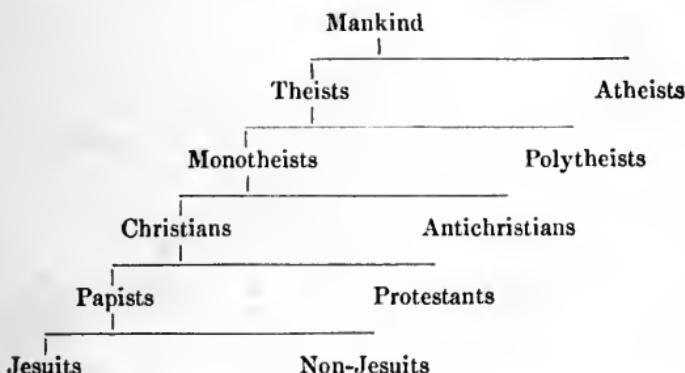
5. Change the following tree into a scale or ladder :
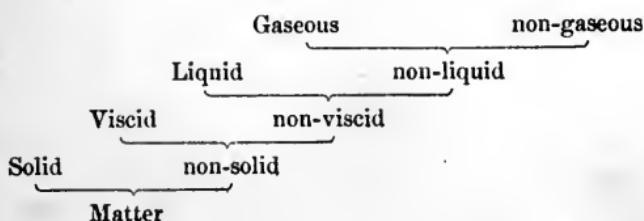
6. Describe the divisions exhibited in the **following** horizontal tree in ordinary language (cf. § 21):

$$\text{Knowledge is....} \begin{cases} \text{Obscure} \\ \text{Clear....} \begin{cases} \text{Confused} \\ \text{Distinct....} \end{cases} \begin{cases} \text{Inadequate} \\ \text{Adequate} \\ \text{Intuitive} \\ \text{Symbolic} \end{cases} \text{Perfect.} \end{cases}$$
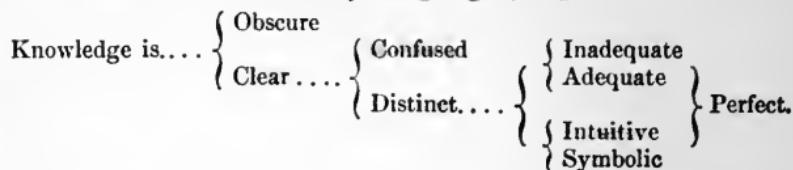
7. Try to divide and subdivide *triangle* so as to include, without cross division (§ 33), the *right-angled*, the *equiangular*, the *obtuse-angled*, and the *isosceles*.

8. Change the following series of definitions, omitting the specific differences and supplying negative members, into a series of divisions, presenting a *scala*, as in Ex. 4:

A carnivore is a flesh-eating mammal.

A mammal is a vertebrate suckling its young.

A vertebrate is an animal having an internal skeleton.

An animal is a sentient organism.

An organism is a living being.

9. Change the following series of definitions, omitting and supplying as above, into a series of divisions, presenting a horizontal logical tree, as in Ex. 6:

Wealth is things useful and agreeable, acquired by labor.

Capital is wealth destined to reproductive consumption.

Circulating capital is capital consumed in a single use.

Wages is circulating capital paid in remuneration of labor.

10. Exhibit the following logical distribution of the sciences in the manner exemplified in § 23:

All rational knowledge, or philosophy in its widest sense, is either *a posteriori* and empirical, or *a priori* and pure. Empirical knowledge gives rise to abstract science, as mathematics; and to concrete science, as the inductive sciences. Pure knowledge, or philosophy in its restricted sense, is either formal or material. Material philosophy, or metaphysics, has two branches—a metaphysic of nature or physics, and a metaphysic of morals or ethics. The chief sciences strictly formal are philology and logic.

VI.—PREDICATION

§ 50. To predicate is either to affirm or deny one
notion of another. Since pure logic does not at all
consider the matter of propositions, but only their
form (§§ 4, 12), it is permissible, according to the
Law of Identity, to affirm any notion of any other,
provided they be not contradictories (§ 8). Thus
there is no logical or formal fault in saying: *The
moon* is *made of green cheese;* or, *The earth* is *a
cube;* or, *Every man* is *dishonest.* However false
to fact such statements may be, they are not logi-
cally absurd; that is, in themselves essentially con-
tradictory. It is possible to entertain the thought.
But to speak of *a spherical cube*, or of something
better than the best, or of *a complete vacuum*, or of
our mutual friend, or of *tempting providence*, or to
say that *all men* are *liars*, is logically absurd, since
a contradiction is involved, and it is not possible to
entertain a thought thus qualified (§ 9). In consid-
ering predication, then, we are not at all concerned
with the material truth or falsity of what is said,
but with the form of the saying, which is limited
only by self-contradiction.

§ 51. Predication is either positive or negative.
It is the issue of comparison. Two notions com-

pared are apprehended as similar or dissimilar, and the judgment pronounces that they agree, or that they disagree. Positive predication affirms or posits, by the Law of Identity, that the subject and predicate are in the relation of part and whole, contained and containing. Negative predication denies or sublates, by the Law of Contradiction, such relation, excluding subject and predicate each from the sphere or comprehension of the other. By the Law of Excluded Middle, no third form of predication is possible; the relation in question between subject and predicate either does or does not exist, it is yea or nay. The ground of this division of the forms of predication is called their Quality; that is, judgments, with reference to their quality, are positive and negative.

§ 52. When one notion is predicated of another, the existence of their objects is neither posited nor sublated. To affirm that *sea-water* is *salty* does not posit the existence of *sea*, or of *water*, or of *salt*, or of the mark *salty*. This is presumed in case of each, the result of prior thought; or the affirmation is conditioned on their existence, thus: *If there* be *sea-water*, *it* is *salty*. Obviously to deny one notion of another does not sublate the existence of their objects. So far of absolute existence. Their relative existence is predicated, conditioned on their absolute existence; that is to say, if they absolutely or really exist, a relation between them is affirmed or denied as existing.

But very often there is occasion to predicate absolute existence. This is accomplished by existential forms of speech or propositions. Thus, *I am* means *I exist, I am existing,* or *I am a being.* The predicate in such case is the summum genus, or its single simple mark. So, also, *Chance is not;* that is to say, there is no such thing. Existential propositions frequently take an inverted form, the place of the transposed subject being occupied by a meaningless particle; as, *It is fine weather; There are not many wise.* Some predications may be construed as existential or otherwise. Thus the latter example may be construed either as, *Not many wise are,* or as, *The wise are not many.*

§ 53. Negative forms call for special remark. A negation strictly pure merely denies one notion of another, no more. If we say, *Smoke is not vapor,* the thought is that these two notions, though liable to be confounded, are so essentially unlike that they should be set entirely apart. This is simply a holding back from error. In other negations there is a thought of a genus which is denied to the subject; as, *Smoke is not a gas;* that is, the genus *gas* does not contain under it *smoke* as one of its kinds. Thirdly, there may be a reference of both notions to a containing genus. Thus in *Men are not brutes,* the thought is limited by the universe *animal,* under which *man* and *brute* are specific contradictories (§ 9). Lastly, the notions may be disparate, or contrary, and as such be denied of each other (§ 31).

A proposition whose predicate is a pure negative is called infinite. If we say, *The soul* is not *mortal*, by this denial we merely ward off error. But if we say, *The soul* is *non-mortal*, as to logical form we affirm, and thereby place *the soul* in the infinite sphere of *non-mortal* beings. This sphere, obtained by the subtraction of *mortals* from the infinite sphere of *beings*, though limited thereby, is still infinite.

Very many notions formally negative have nevertheless a positive character (§ 30). Each of these is usually thought as a finite mark or universe. Such are the notions *helpless, unpleasant, unwell, infamous, uneven, immortal*. Thus, if we say, *The soul* is *immortal*, there is affirmed of it, besides the negative notion of infinity, the positive one of continuous existence. This marks a definite genus, quite distinguishable from *non-mortal*.

§ 54. Predication is either in the intensive or in the extensive whole (§ 23 sq.). The distinction is grounded on the relation of subject and predicate, as reciprocally whole and part.

In an intensive judgment, the subject is the whole or major term, the predicate is the part or minor term. Thus, in the attributive judgment *The earth* is *spherical*, the notion *earth* is an intensive whole consisting of a complement of marks, and the mark *spherical* attributed to it enters into or is recognized as a part of this whole, it being only one mark out of many that characterize the notion *earth*. This

form is conventionally interpreted, *The earth* comprehends *spherical*.

In an extensive judgment, the predicate is the whole or major term, the subject is the part or minor term. Thus, in the proposition *The earth* is *a sphere*, the notion *sphere* is an extensive whole, a genus, constituted of many kinds of things, as the other planets, their satellites, the sun, the geometrical sphere, globular fruits, rain-drops, etc. Now, *the earth* is declared to be one of the many things denoted by *sphere*, to be a part of this whole, a member of the genus. This form is conventionally interpreted, *The earth* is contained under *sphere*.

Consequently, while a qualitative judgment may have an individual as its subject, it cannot have an individual predicate. For the predicate in intension is a mark, in extension a genus (§ 20); an individual cannot be either. We may say, *Great* is *Diana*, but this is a rhetorical inversion; *Diana* is the subject, and the predicate is *great*. We may say, *The rival of Plato* is *Aristotle*, but this is not qualitative, but a quantitative equivalent proposition.

§ 55. The ten categories or predicaments of Aristotle, about which opinions greatly differ, are as follows, illustrated by his own examples:

1. Substance;—it is a man, a horse, etc.
2. Quantity;—it is two cubits long, three, etc.
3. Quality;—it is white, grammatical, etc.
4. Relation;—it is half as large, greater, etc.

5. Action;—it cuts, burns, etc.
6. Passion;—it is cut, is burned, etc.
7. Place;—it is in the Agora, the Lyceum, etc.
8. Time;—it is to-day, was yesterday, etc.
9. Posture;—it is reclining, seated, etc.
10. Possession;—it is having shoes, armor, etc.

These may be interpreted as an exhaustive series of summa genera standing next the true summum genus, *Being;* metaphysically, a classification of the modes of objective or real existence; logically, a classification of the most general notions that can be predicated of any subject. That is to say, anything whatever may be said to be *a substance, a quantity,* or some other one, at least, of these highly generalized notions. In this view they are first intentions or names of things (§ 4).

But it seems clear, both from the title of the list and from his examples, that Aristotle intended to name rather all the possible forms under which things may be represented in thought. Thus, if we say of anything, *It* is *a man,* we are thinking it in the category of substance; or, if we say, *It* is *seated,* we are representing it in the predicament of posture; and so any judgment whatever will fall into one or another of these ten formal categories. In this view they are second intentions or names of forms of thought.

§ 56. Having settled the category in which a predication places the subject, it may be asked, What kinds of predicates are then possible; or, in

other words, what are the second intentions or forms of its possible predicates? The answer is Aristotle's doctrine of the predicables, as follows: Every judgment affirms or denies of its subject one or another of these four relations:

1. Definition; as, Man is a rational animal . . . . } = All of the essence } Convertible.
2. Property; as, Man is risible. . . . = None of the essence)
3. Genus; as, Man is an animal . . = Part of the essence } Inconvertible.
4. Accident; as, Man is a biped . . = None of the essence)

It has been proposed to substitute specific difference for definition, since it already contains genus, and to make the number five by adding species as predicable of individuals. But the list would not be improved; for, as Aristotle himself remarked, both difference and species are of the nature of genus, and interchangeable with it (§§ 27, 37).

§ 57. **Praxis.** Write the quality of each of the following propositions, stating whether the predication is logically permissible or not, and why, and noting existential forms:

1. I do not just now remember anything I have forgotten.
2. A national debt is a national blessing.
3. There is none that doeth good, no, not so much as one. Let there be light; and there was light.
4. A flying arrow is at rest.
5. It is impossible to love and be wise.
6. There is a tide in the affairs of men
   Which, taken at the flood, leads on to fortune.
7. I think there be six Richmonds in the field.

5

8. Man is not a beast for burdens, nor a reptile for bruising.

9. There is no place like home.

10. Let us try to amuse ourselves by doing nothing, and so making ourselves miserable.

11. An idiot is irrational.   A brute is non-rational.

Note which is the major term in the following, with the reason:

12. Pearls are precious.   Rubies are stones.

13. Heresy is sin.   Solomon is wise.

Predicate a categorical class of each of these subjects:

14. Gold.   The wealth of Crœsus.   Antiquity.   Red. Parallelism.   A battle.   New York.   A multitude. Upright.   A defeat.

State to what category and to what predicable each of these judgments belongs:

15. Snow is frozen mist; it falls lightly; is very white; but is easily discolored; it is colder than water; lies level; occurs only in winter; but not at the equator; it has minute crystalline forms; and accumulates in huge masses.

§ 58. As a product of thought, a judgment is the result of comparison. Two notions are compared, and the judgment pronounces that they agree or disagree. In case they disagree, they are set apart by a denial. In case they agree, they are unified by an affirmation. To judge affirmatively is to bring a notion into or under another. One is thought as determined by the other; for either the latter is brought as a mark into the one, which is thereby determined, or else the one is brought under the other as a class, and thereby determined. A judgment expressed in words, since it is placed before us for acceptance, is called a proposition. What is subjectively a judgment is objectively a proposition.

The propositional forms with which logic is immediately concerned are the conditional and the categorical. A conditional proposition states a comparison so nearly complete that only some provision remains in question. The contingency is expressed as a condition, thus: *If air be pure, it is wholesome.* Categorical propositions constitute the negative member of the dichotomy. A categorical proposition is one wherein no contingency or condi-

tion is expressed. This difference is obviously not essential; but since the conditional declares relatively to some provision, and the categorical names none, the latter is said to declare absolutely. In strictness, however, all propositions, except axioms, are conditioned on prior thoughts, and on the existence of their objects (§ 52). The provision may or may not be expressed. While, therefore, we shall give the conditional form special consideration in a subsequent chapter (§ 110 sq.), we shall not care to exclude it meantime from view, though our attention for the present will be directed chiefly to the categorical form.

§ 59. The categorical proposition is severed by partition into three portions (§ 24). In affirmation these are:

1st. The notion of something determined, called the Subject.

2d. The notion of something determining, called the Predicate.

3d. The part which expresses this relation, called the Copula.

In the negative proposition there is no determination of one notion by another. But in both forms something is spoken of, which is the Subject; something is said of it, which is the Predicate; and that which says this is the Copula. Thus, *Snow* is *Pure*, or S is P. In early logic the predicate includes the copula, and this is still the usage of grammarians. But logicians now reckon the copula as a

distinct co-ordinate part. The subject and predicate, being the extremes of the partition, are called the Terms of the proposition. It is not at all requisite that a term should consist of a single word; each term may be composed of many words in intricate grammatical relations. E. g.,

> " With taper light
> To seek the beauteous eye of heaven to garnish (=*subject*)
> Is (=*copula*) wasteful and ridiculous excess " (=*predicate*).

§ 60. A judgment always expresses the relation of two notions now in mind; therefore the copula must always appear as the present tense of the verb *to be: For the mind is its own kingdom, in which an eternal now does always last.* Very often in common speech it is absorbed in verb forms, or elided, and a whole proposition may be expressed by a single word. E. g., *Stars twinkle,* i. e. *Stars* are *things that twinkle; He loved,* i. e. *He* is *one who loved; Cogito,* i. e. *I* am *thinking; Ilium fuit,* i. e. *Troy was,* i. e. *Troy* is *something that formerly existed* (existential); *Did he say so?* Ans. *Yes,* i. e. *He* is *one who said so.* All verbs are perhaps fundamentally one, the verb *to be* of the summum genus *being,* their variety arising from the incorporation of various temporal and attributive notions with this simple verbal element, its own past and future forms being adverbial notions incorporated with its present tense.

The copula admits of only one qualification, negation. Hence in a negative proposition the nega-

tive particle, wherever it may occur, is a part of the copula. E. g., *The quality of mercy* is not *strained;* No *chastisement* is *joyous;* Not *a drum* was *heard;* Not *every mistake* is *culpable; Britannia needs* no *bulwark*, i. e. *Britannia* is not *needing a bulwark.*

Let it be observed that affirmative propositions often contain negatives in the subject or in the predicate, and should not be mistaken for negative propositions. E. g., *To wonder not* is *a rare art; Axioms affirm what no one can deny.* Also observe that propositions affirmative in form are sometimes negative in thought. E. g., *The brute perishes; He* is *blind; Darkness and silence fall on land and sea.* Negative thought may also be conveyed in affirmative forms by means of such words and phrases as *without, beyond, far from, the reverse of, on the contrary, wanting, deficient in, devoid of,* and the like. E. g., *We can do without it.*

§ 61. In accordance with its postulate (§ 13), logic requires that all propositions shall be transformed, as has been shown, so that, without addition or retrenchment or distortion of the thought, the three parts, subject, copula, predicate, shall severally appear. The process is sometimes quite troublesome, and the result awkward, but it is nevertheless indispensable. E. g., *So he said* becomes *What has just been said* is *what he said; If he should come to-morrow, he will probably stay till Monday* becomes *The happening of his arrival to-*

morrow is *an event from which it may be inferred as probable that he will stay till Monday.*

The proposition often exhibits rhetorical inversions, and a displacement of minor parts. E. g., *Great* is *Diana of the Ephesians ; Few and short* were *the prayers we said ; Flashed all their sabres bare ; Gold and silver have I none ; but what I have, that give I thee.* Herein order must be restored, the subject naturally coming first.

All inversions and displacements corrected, all elisions supplied, and the three parts stated distinctly in their natural order, constitute the reduction of a proposition to its strict logical form. Hence every proposition must, for logical purposes, be reduced to one or the other of the two invariable forms, S is P, or S is not P.

§ 62. It has already been stated that propositions, as to their Quality, are positive and negative (§ 51). It is now to be observed that propositions, as to their Quantity, are total and partial. The quantity of a judgment or proposition is determined solely by the quantity of its subject, according as this is definite or indefinite. The following scheme exhibits this division with subdivisions :

Propositions
- Total, definite . . .
  - Individual, as, All the world's a stage.
  - Universal, as, All men are players.
- Partial, indefinite
  - Divisive, as, Some play soldier.
  - Indivisive, as, Some act armies.

The quantity of the subject, and hence of the proposition, is indicated by the predesignation *all* or

some, or its equivalent. These two exhaust the possibilities of predication; that is to say, every possible proposition predicates either concerning *all*, or concerning *some*, of its subject.

It is often the case that no sign of quantity is prefixed. A judgment always has quantity in the mind of the thinker and speaker, but the hearer may be left to surmise the quantity from the matter or the context. E. g., *Birds breathe*, i. e. all do, the predicate being of the essence; *Birds sing*, i. e. some do, the predicate being an accident. On reducing such propositions to strict logical form, it is generally needful to designate the quantity by its sign.

§ 63. Individual propositions are those in which the subject is thought as an indivisible total. The subject may be a proper noun, as in *Cæsar* is *ambitious*; or something designated by the definite article, or any demonstrative or possessive, as in *The world* is *round*, *This man* is *crazy*, *Let your words* be *few*. It may be a collective whole, as in *The college of apostles* was *typified in the twelve tribes* (§ 24). It may even be a genus, as in *The horse* is *a noble animal*. It may be unified by *all*, as in *All Jerusalem went out to meet him*.

§ 64. Universal propositions are those in which the subject is thought as a divisible total. The subject is said to be distributed, because what is said of it as a whole is thought as distributively

applicable to each part, as in *All men* are *players*,
i. e. all without exception; and in *Every man* is *a
player*, i. e. each taken severally. So also in *No
man* is *perfect*.

Predesignations or signs of universality or dis-
tribution are *all*, *a* or *an* or *every*, *each*, *both*, *neither*,
not any, *none*, *always*, *never*, *whoever*, etc.

It appears that *all* is ambiguous, meaning either
all as an undistributed unity, or all as a distributed
plurality, as in *Drink ye all of it*. The matter
generally determines whether the meaning is cu-
mular or distributive.

Names of substances, as *water, flesh, flame, iron;*
of forces, as *gravity, heat;* of actions, as *to walk,
talking;* and abstract terms, as *charity*, are usually
universal without predesignation.

§ 65. Partial or indefinite propositions are those
in which the subject is thought as less than the
whole denoted by its naked form. We do not
think definitely of all, but partially or particularly
of some. The indefinite *some*, as the predesigna-
tion of logical quantity in affirmative propositions,
means *at least some, perhaps all;* in negatives, it
means *at least some not, perhaps none.* E. g., *Some
men play soldier*, i. e. at least some, perhaps all, do;
Some men are not *pure*, i. e. at least some are not,
perhaps none are. A subject thus quantified is said
to be undistributed.

Predesignations or signs of particular or indefi-
nite propositions are *some, a few, a* or *an* or *one, two,* .

three, etc., *certain, there are—that, not all, not every, sometimes, somewhere*, etc. E. g., *A few* are *saved,* i. e. some, a small number, are, perhaps all; *There* are *men that practise self-denial*, i. e. some exist, at least a few, perhaps many, perhaps all; *All* are *not here*, i. e. some are not here, contradicting *All* are *here ; Not every step counts*, i. e. some steps do not count. The sign *any* signifies an indifferent some ; the statement is limited to some, but is applicable to every one, as in *Anybody can do that.*

There are signs that approximate the total, but being less are still particular, as *many, most, almost all, the large majority of*, etc. Others are nearly total negatives, as *few, very few, hardly* or *scarcely any, little, small, slight, rare, seldom*, etc. E. g., *Many* are *called, but few chosen*, i. e. at least some, nearly all, perhaps all are called, but at least some are not, nearly none, only a few, chosen ; *Few* are *saved*, i. e. many are not, perhaps none ; *Little reck I*, i. e. almost and may be not at all. Let it be remarked that *a few* and *a little* are affirmative, and that *not all*, when cumular, is total.

§ 66. It appears that *some* also is ambiguous. Besides the divisive sense just considered, it has an indivisive, undistributed sense. E. g., *Some men act armies*, means that a portion, a section, wholly indefinite as to extent, do so. It has the same meaning in : *Give me some water ; Some time has passed ; We have come some distance ; Some of the way was rough ; Some of the company is in ad-*

vance. The thought is of a mathematical quantity, of a mass, logically indivisible (§ 24).

Besides its indefinite significations, *some* is often used in a semi-definite sense, meaning *some at most, not all,* as in *Some men seek fame,* i. e. some at most do, but not all, for some do not. This form, however, is compound, and will be examined in the next chapter (§ 72).

§ 67. Combining the quality and quantity of propositions, and symbolizing by vowels, we have the following scheme of the four simple propositional forms :

Quantity.	Quality.	Symbol.	Formula.	Example.
Universal Affirmative,—	A—	All S is P........		All oaks are exogens.
Universal Negative, —	E—	No S is P........		No oaks are vines.
Particular Affirmative,—	I —	Some S is P......		Some oaks are trees.
Particular Negative, —	O—	Some S is not P...		Some are not shrubs.

The examples are in strict logical form. Individual propositions (§ 63), since the subject is definite, are treated as universals, and symbolized by A and E. Propositions whose subject is the indivisive *some* (§ 66), being indefinite, may be symbolized by I and O.

§ 68. A simple proposition comprises only one judgment ; it contains not more than one subject and one predicate. It may, however, consist of many grammatical elements ; as, *Well-organized and skilfully administered governments* are *productive of happiness in their subjects.*

A complex proposition involves with the principal judgment one or more subordinate judgments. This subordinate element appears as a clause incidental to the principal subject or predicate. E. g., A man *who is learned* is respected; I am monarch of all *I survey;* They *that are wise* shall shine *as the stars* (*shine*).

A subdivision may be made into explicative and restrictive clauses. The explicative clause merely unfolds the marks connoted by the notion it qualifies; as, Man, *who is born of woman,* is of few days. The restrictive clause limits the notion it qualifies; as, Ill blows the wind *that profits nobody.* Not all winds, but all in a limited class, are here spoken of. The concessive clause removes a conceivable restriction; as, I will trust him *though he slay me.* When a restriction is a condition, the categorical may be transformed to a conditional proposition; thus, A man *who is learned* is respected, becomes, If a man be learned, he is respected.

Now, be it observed that the complex proposition is treated logically as a simple proposition, the incidental clauses being regarded as mere substantive, adjective, or adverbial qualifiers. In reducing to strict logical form, it is needful to subordinate clauses to the principal subject and predicate, and to place them in close connection with the notions they qualify; as, He *who, though he be rich, is saving* is one *that can share* with him *who is needy* without lessening *what is enjoyed;* here the form is simply, *S* is *P.* Indeed, the complex proposition

is often directly reducible to one that is strictly simple; thus, The man *who is learned* is respected, becomes, The learned man is respected.

This account of the complex proposition is given to prevent clauses from being mistaken for principal propositions, and so confusing it with the compound proposition.

§ 69. **Praxis.** Concerning each of the following propositions, answer, in their order, these five questions: Is it conditional or categorical? If categorical, what is its strict logical form? What is its symbol of quantity and quality? Is it individual? Aside from its form, is it essentially positive or negative?

1. No man is wiser because of his learning.
2. The senate has adjourned.   Charity affords relief.
3. If you understand this, you can explain it.
4. From peak to peak the rattling crags among
   Leaps the live thunder.
5. There was a sound of revelry by night.
6.   This to hear
   Would Desdemona seriously incline.
7. Not to know me argues yourself unknown.
8. I implore you, reject not this bill.
9. The blind are helpless.   Richard III. was infamous.
10. Men are all sinners.   No news is good news.
11. Few patriots are disinterested.   Diogenes was no fool.
12. All these claims upon my time overpower me.
13. Virtue is teachable, if it be knowledge.
14. Not many, if any, metals are without lustre.

15. Not being rich is not always an evil.

16. Hardly any virtue is safe from passing into vice.

17. Those here present constitute the class in logic.

18. There's few or none do know me. That horse won the race. A general cry was, No surrender.

19. There are who ask not if thine eye be on them.

20. I will not let thee go, unless thou bless me.

21. The most skilful general was Napoleon.

22. Little preface is needed. Not every man is honest.

23. The circle is the figure of greatest area.

24. Some education is desirable. Some pudding, a slice, if you please. Some love to roam.

25. All honorable conduct is not to be rewarded.

26. Yonder forest is a covert for outlaws.

27. Gold is precious. To converse is pleasant.

Reduce each of the following propositions to strict logical form, and affix its symbol:

28. Nothing is harmless that is mistaken for virtue.

29. One truth is clear, whatever is, is right.

30. All is not gold that glitters.

31. He who truly loves most is not he who flatters.

32. Though this be madness, yet there's method in it.

33. Even a fool is counted wise, when he holdeth his peace. That I am is no proof that he is.

34. They strive that they may enter in.

35. There's a divinity that shapes our ends,
Rough-hew them how we will.

VIII.—COMPOUND PROPOSITIONS

§ 70. A compound proposition is one that comprises two or more judgments, co-ordinate or nearly so. For logical treatment the components are to be separated, and stated independently. Such propositions are of two kinds, according as the composing elements are more or less obvious.

The first kind, wherein the components are quite obvious, has received no specific name, and needs only a few illustrations. E. g., *Art is long, and time is fleeting; Every man desireth to live long, but no man would be old.* In *Veni, vidi, vici,* there are three propositions. So also in *Pompey, Crassus, and Cæsar were triumvirs.* It is often the case that a simple proposition has a compound subject or predicate, as in *Pompey, Crassus, and Cæsar were the triumvirs,* for the three are here taken collectively as one whole. So *Roses and lilies contend for a home in her cheek* is single and simple; but in *Darkness and silence settle on land and sea* there are four propositions.

§ 71. Compound propositions of the second class, having components less obvious, require analysis, and are called exponibles. They are chiefly exclusives and exceptives.

Exclusives may be formulated and exemplified thus:

$$\text{Only A is B} = \begin{cases} \text{A is B} \ldots\ldots\ldots\ldots\ldots\ldots & \text{A or I} \\ \text{Non-A is not B} \ldots\ldots\ldots\ldots & \text{E or O} \end{cases}$$

$$\text{E. g., Faith alone justifies} = \begin{cases} \text{Faith justifies} \ldots\ldots\ldots\ldots & \text{A} \\ \text{What is not faith does not} \\ \quad \text{justify} \ldots\ldots\ldots\ldots\ldots \end{cases} \text{E}$$

It is evident that this proposition may be inverted and the excluding particle made to appear in the predicate; thus, *Justification is by faith alone*, i. e. *B is only A.*

Exceptives are exemplified in *All but one were saved*, which means *Nearly all were saved*, and *One was not saved*, I and O. It should be noted that *but* is sometimes not exceptive, but merely adversative, as herein; also, that it sometimes means *that—not*, as in *It cannot be but nature hath some director*.

No useful rule can be given for the resolution of these exponibles. The components differ in quality, and one is direct and the other implied. But the distinction between exclusives and exceptives is of no logical moment, for they are mutually convertible, the difference being that what is the direct judgment in the one form becomes the indirect in the other.

The following are some of the exclusive and exceptive particles: *only, alone, merely, solely, save, but*, etc. These particles, when qualifying a universal subject, quantify the predicate totally; as, *God alone is wise*, i. e. He is all the wise. When qualifying the entire predicate, they limit the sub-

ject to that predicate; as, *The sacraments are but two,* i. e. there are no more. Sometimes the exclusion or exception is in the sense, and not expressed; as, *(There is only) one Lord, one faith, one baptism.*

§ 72. Our thoughts are often incompletely expressed. A proposition may be accompanied by another limiting judgment unexpressed, with no sign of implication, yet understood, because of our knowledge of the matter. Thus if we say, *Some flowers are fragrant,* knowing very well that some are not, the proposition is accompanied by the counter-judgment, *Some flowers are not fragrant.* If this double thought be completely expressed in a single, grammatically simple proposition, it is, *Only some flowers are fragrant.* Now, this form is an exponible, a logical compound, which analyzes into the two logically simple propositions:

Some flowers (I know not how many) are fragrant.... I
Some flowers (I know not how many) are not fragrant...... ⊙

Each of these, considered in itself, entirely apart from the other, is wholly indefinite; for the meaning of *some, I know not how many,* must in that case be *at least some, perhaps all.* It is evident, then, that *some,* in the semi-definite sense of *some at most, not all* (§ 66), is equivalent to *only some,* and does not occur unless one judgment is thought as limiting another. Therefore, propositions quantified by the so-called semi-definite *some* are compound propositions.

6

Since logic proposes to exhibit a thorough analysis of thought, it should in no case stop short of simple forms. It is out of character to present compound forms as the result of analysis, and especially to rank them as co-ordinate with simple forms. Hence the semi-definite proposition must be denied a position among the elementary forms, and assigned a place among the abbreviated, elliptical modes of statement, subject to analysis and full, discrete expression.

§ 73. The predicate of a simple qualitative proposition has no quantity whatever. We mean to say, not merely that it may have none, but that it cannot possibly have any. This is quite obvious in case of intensive propositions. E. g., *An athlete is strong*, cannot mean either *all strong* or *some strong*, which is senseless, but simply that the mark *strong* is found in the subject. In case of negative propositions, both intensive and extensive, the same is true. E. g., *Some athletes are not studious*, or *are not students*, means simply to deny the notion *studious* or *students* of the subject, without any quantification of *students*. The subject and predicate are merely coexclusive. The same is equally true, though perhaps not so clearly evident, in case of affirmative propositions in extension. E. g., *All athletes are sportsmen*, or *Some athletes are snobs*, merely places *athletes* in a class, without any thought whatever of the quantity of the class; that is, without thinking it as either *all* or *some*.

The statement that a predicate cannot have quantity is true of simple qualitative propositions only. We can readily, and often do, think quantity into the predicate of extensive propositions; but this is to compound our thoughts, the predicate becoming for a moment the subject of thought, and then being restored to its place quantified. For example, if we say, *All triangles are trilaterals*, and then think that *All trilaterals are triangles*, we may express the double thought directly thus: *All triangles are all trilaterals*. It has already been remarked that an exclusive or an exceptive added to a universal subject quantifies the predicate totally (§ 71), and so the same thought may be indirectly expressed by the exponible, *Only triangles are trilaterals*. In like manner predicates of other forms may be quantified.

A very convenient mode of symbolizing the forms of such propositions has been devised, which we shall have some occasion to use. Let **a** stand for a total, **i** for a particular term; also let **f** represent the affirmative, and **n** the negative copula. Then the forms spoken of may be represented thus:

All triangles are (all) trilaterals....*all* are *all*..........**afa**
All triangles are (some) figures....*all* are *some*........**afi**
Some men are (all) priests.........*some* are *all*........**ifa**
Some men are (some) poets........*some* are *some*.......**ifi**
Some men are not (any) poets.....*some* are not *any*....**ina**
No oaks are (any) vines...........not *any* are *any*....**ana**

§ 74. Two views may be taken of these forms.

With reference to their origin, they are compound propositions, formed from components, into which they can be resolved. In this view, they cannot be allowed co-ordinate rank with the four simple forms (§ 67), and must be held subject to analysis.

If viewed in themselves, without reference to their origin, they are seen to be propositions, not in the qualitative, but in the quantitative whole (§ 23). For consider the meaning of **afa**. Take, *All men are all bimana.* Here the ambiguous *all* (§ 64) has changed from the distributive *all*, which quantifies the components, to the cumular, indivisible *all* (§ 63). It does not mean, *Every man is every bimana,* which is nonsense, but *All men* (taken together as a mass) = *all bimana* (taken together as a mass). The proposition, considered in itself, is therefore a mathematical equation. This is clearly true of the affirmative forms. As to negative forms, *any* thought into the predicate does not properly quantify it, but serves rather to emphasize the negation, the proposition remaining qualitative; though, of course, a mass may be denied of a mass, and we may think a negative proposition in either whole.

While, therefore, a simple qualitative judgment always has a quantified subject, it cannot have a quantified predicate. It follows, also, that a system of logic built upon the quantification of the predicate is vicious, either by confusing compound with simple forms, or else by unnaturally transferring all thought to the quantitative whole, and so making logic merely a branch of applied mathematics.

§ 75. Finally, it is to be observed that in drawing inferences, and so transforming thought, it is frequently necessary to make a predicate, for a moment at least, the subject of a judgment to which quantity is assigned. Then this is spoken of as the quantity of the predicate, and must be taken into account in many forms of illation. Now, this so-called distribution of the predicate, ascertained by compounding the thought, and having no verbal sign, depends on the quality of the judgment, and is expressed by the following RULE: **Negatives distribute the predicate, affirmatives do not.**

In view of the preceding discussion, it is clear that this statement taken in itself as to simple judgments is not true. But taken merely as a derived rule to be applied in illation, tersely and hence imperfectly expressed, it serves, under the given explanation, as an unerring guide in logical processes.

§ 76. **Praxis.** Referring to each of the following propositions by its number, state whether it be simple, complex, or compound. If either of the two former, reduce it to strict logical form, and affix the capital symbol (§ 67). If compound, state what kind, then write its components, affixing to each the capital symbol, and noting the semi-definite *some*. If its predicate have a sign of quantity, symbolize literally (§ 73), then state its components with their capital symbols.

1. None but the brave deserve the fair.
2. Mercy but murders, pardoning those that kill.

3. Men may come, and men may go,
   But I go on forever.

4. Length, breadth, and depth are all the dimensions of extension.

5. When I was a boy, I always chose the wrong side.

6. It is the duty of every man to fear God and honor the king.

7. Jonah sought to evade the God who is omnipresent.

8. Few, few shall part where many meet,
   The snow shall be their winding-sheet.

9. There is no fireside, howsoe'er defended,
   But has one vacant chair.

10. Not every one that saith unto me, Lord, Lord, shall enter in, but he that doeth the will of my Father.

11. Some inspired men were all of the apostles.

12. Brutus, in killing Cæsar, was merely patriotic.

13. The moon is only our satellite.

14. The moon is our only satellite.

15. The paths of glory lead but to the grave.

16. A fool thinks none except himself wise.

17. Ho! hearts, tongues, figures, scribes, bards, poets, cannot
    Think, speak, cast, write, sing, number—hoo!—
    His love to Antony.

18. Live how we can, yet die we must.

19. Some who are poor are nevertheless contented.

20. All grace is all free favor.   Certain gifts are not any favor.

21. All present are some of my friends.

22. My tasks are all but impossible.

23. The quarrel toucheth none but us alone.

24. Whereto serves mercy, but to confront the visage of offence?

I.—IMMEDIATE INFERENCE

§ 77. A judgment either affirms or denies that one notion is in or under another (§ 58). A division of judgments, grounded on the process by which they are formed, is as follows :

Judgments are
$\begin{cases} \text{Intuitions.} \\ \text{Inferences.} \end{cases}$
$\begin{cases} \text{Inductive.} \\ \text{Deductive.} \end{cases}$
$\begin{cases} \text{Immediate.} \\ \text{Mediate.} \end{cases}$

Intuitions are self-evident, axiomatic judgments. They are the origin of all knowledge, and determine all other judgments; for example, the Primary Laws (§ 7 sq.).

Inferences are enunciations in which from something laid down and admitted, something distinct from what is laid down follows of necessity. Or, more simply, to infer is to derive a judgment from one or more premised judgments. Both the process and the conclusion are called inferences.

Inductive inferences are universal judgments derived from particular cases, and furnishing premises for subsequent deduction. The general definition of Logic includes them (§ 6), but the present work

is limited to deductive inferences. Inductive Logic requires a separate treatise.

Deductive inferences are judgments of equal or less generality than the premises from which they are derived. They are especially the subject of Deductive Logic, and are of two kinds, immediate and mediate.

When two notions having a given relation are concluded of each other in another relation without the introduction of a third notion, the inference is immediate. In this case one judgment is derived directly from another. The conclusion has but one premise, the given judgment. The matter in both is the same; the relation is modified.

A mediate inference or a reasoning is accomplished through a third notion used as a medium of comparison. It has two premises.

§ 78. Implications should be distinguished from inferences. An implied judgment is one actually coexisting with the given judgment, either merely in thought or involved covertly in the expression. An inferred judgment is one that only potentially exists in the given judgment, and may be derived from it. The statement of the one is nothing new, there is no advance, no progress of thought, but only its full expression; that of the other contains something new, there is a step forward, a progress of thought.

The forms extension and intension are hardly to be considered even as implying each other, much

less as inferable one from the other. They are simply different aspects of the thought, which necessarily coexist, one having merely accidental preponderance (§ 20).

Correlatives merely imply and are not inferable from one another (§ 30). To infer from *The cause of the explosion was a spark*, that *The effect of a spark was the explosion*, is a fallacy (§ 146).

The interchange of active and passive forms is not an inference. *God made the world*, and *The world was made by God*, imply each other, or rather are equipollent (§ 13).

Incomplete speech implies thought, as in the semi-definite proposition (§ 72). Thus if we say, *Some men are rich*, it is accompanied by the judgment that *Some men are not rich*. But this actually coexistent thought is not inferred. We cannot say that from *Some men are rich*, it follows that some are not. An exponible contains an implied, indirect judgment covertly expressed, as in *Only some men are rich*.

Finally, neither the compounding of two or more simple propositions, nor the resolution of a compound proposition into its components, should be mistaken for inference.

§ 79. Also preparatory to an account of the several kinds of immediate inference for which we shall have subsequent use, we state a prohibition applied to all deductions in the following RULE: **The quantification must not be increased.**

Truly we may deduce *all* from *all*, *some* from *some*, *some* from *all*, but not *all* from *some*. It is evident that what is said only of *some* furnishes no ground for a deduction concerning *all*. The attempt to make this deduction in violation of the foregoing rule is called *the illicit process*. The principles of induction license the inference of *all* from *some*, and so exhaust the possible processes.

§ 80. **Determination.** Immediate inference by determination is one of the four kinds to be noted. The same mark may be added to both terms of a proposition, by which they are more closely determined (§ 42). The new judgment thus formed is an immediate inference from the given premise. Thus, from *Coal is fuel*, it follows that *Cheap coal is cheap fuel;* if *Science be system*, then *A false science is a false system*. We must be on our guard not to use a determinant ambiguously, as in *A king is a man*, therefore *A good king is a good man*. The narrowing of both subject and predicate by thinking a mark into them is passing from genus to species. We observe that the subtraction of the same mark from both terms is legitimate; but the remainder is an implicit judgment, not an inference.

Inverting the foregoing process, the two terms of a proposition may be added as marks to the same concept. Thus, if *Science be system*, then *A scientific arrangement is a systematic arrangement*. Also two propositions may be combined, the terms

of one being added as marks to the terms of the other. Thus, if *A museum be a collection of specimens*, then *A scientific museum is a systematized collection of specimens*.

§ 81. **Infinitation.** This mode of immediate inference passes from the purely negative to the infinite judgment (§ 53). It places the subject in the outer, infinite sphere. Thus, if *The soul be not mortal*, then *The soul is non-mortal*. These propositions express different thoughts. They are similar, but not identical. The reverse inference is included, for the sake of brevity, under the same name. Also purely affirmative and doubly negative propositions are infinitated; thus, if *Man be mortal*, then *No man is non-mortal*, and reversed. Hence the general RULE: **Change the quality of the judgment, and also of its predicate.** The quantity of the judgment remains unchanged. In using privatives, as *in-*, *un-*, *dis-*, *-less*, etc., we must be on our guard lest we derive too much. With this precaution, we add the following complete series of examples:

Since All metals are fusible,	then	No metal is infusible.....	A yields	E
" No miser is happy,	"	Every miser is unhappy..	E "	A
" Some sins are pardonable,	" {	Some sins are not un- pardonable....... } ...I	"	O
" Some men are not gentle,	"	Some men are ungentle...O	"	I

§ 82. **Conversion.** In immediate inference by conversion, the terms are transposed. Besides observing the general rules already given (§§ 75, 79),

we must take heed to make a total transference; that is, the whole naked subject must be made predicate, and *vice versa*. By naked is meant without the sign of quantity *all* or *some*. Thus, from *Every old man has been a boy*, we cannot infer that *Some boy has been an old man*, but that *Some one who has been a boy is an old man*. Hence it is generally needful before converting to reduce the proposition to its strict logical form.

We shall consider only three kinds of illative conversion, and these only so far as our subsequent need in syllogizing requires, which is that we be able to convert each of the four forms A, E, I, O.

1st. *Simple conversion* transposes the terms without changing the quantity or the quality of the proposition. It is applied to E, and to I; thus:

> No one without sympathies is a true poet;.....E......**ana**
> ∴No true poet is without sympathies...........E

> Some mathematicians are poor financiers;......I.....**ifi**
> ∴Some poor financiers are mathematicians......I

2d. *Conversion per accidens*, or by limitation, reduces the quantity without changing the quality of the proposition. It is applied to A, and the converse is I; thus:

> All plane triangles are rectilinear figures;......A.....**afi**
> ∴Some rectilinear figures are plane triangles.....I

This is called *per accidens* because it is not the transfer of a predicate *per se*, but only of an unes-

sential or accidental part which that term, viewed universally, includes.

Observe that the rule in § 79 forbids retracing the step, reconverting the I into A, which would be the illicit process.

If, on subjecting the predicate of A to inquiry, the proposition is recognized as **afa,** as when a property, a definition, or a division is predicated, then it is convertible simply.

Also E may be converted *per accidens ;* but this case is rather simple conversion followed by subalternation.

3d. *Conversion by contraposition,* or by negation, changes the quality but not the quantity of the proposition. It is applied to O, and the converse is I. To contrapone, we have the following RULE: **Infinitate, and then convert simply.** Thus:

Some pure air is not wholesome;.....O......I.....**ifi**
∴ Some unwholesome air is pure........I

This is evidently a double process. It was devised to convert O, which cannot be converted simply or *per accidens,* as either would be the illicit process. It is applicable also to A.

Upon inspection it is obvious that the whole doctrine of conversion has respect to extension. An intensive judgment cannot be converted. But on changing its predicate to a class notion it becomes extensive, and so convertible.

Since an individual cannot be a predicate (§ 54),

it follows that an individual proposition (§ 63), though it be symbolized by A or E (§ 67), is inconvertible. We say *Juno is a queen*, and may say *One queen is Juno ;* but this apparent conversion *per accidens* is merely a rhetorical inversion (§ 61); the subject of thought is still *Juno*. No mere inversion is a logical conversion.

§ 83. **Opposition.** A proposition in any one of the four forms A, E, I, O, is in opposition to the same matter in each of the other three forms. The relations are such that if the given proposition be formally true or false, we can immediately infer the formal truth or falsity of some of the others. Opposition is of four kinds, exhibited thus:
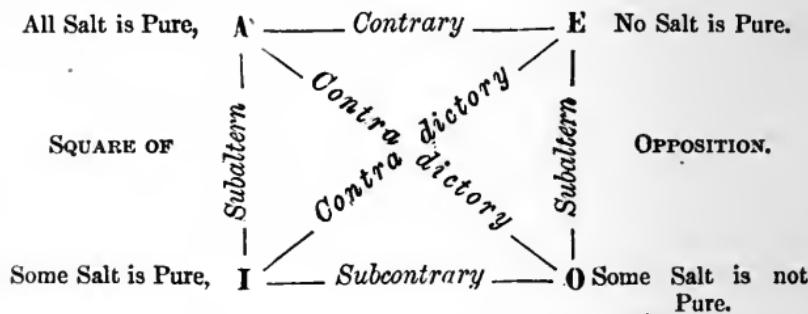
1st. Contradictory opposition exists between A and O, and between E and I, propositions having the same naked or unquantified subject and predicate, but which differ in both quantity and quality. Rule: **Both cannot be true, and both cannot be false**. This is merely a specific statement of the Laws of Contradiction and Excluded Middle (§ 11). E. g., If *All Salt is Pure* be sublated, then

by an immediate inference we can posit *Some Salt is not Pure*. If *Some Salt is Pure* be posited, then we can immediately sublate *No Salt is Pure*, and so on. If it be true that *Every man has a conscience*, then it cannot be that *Some men have no conscience*. Such propositions are said to be diametrically opposed. Contradiction is pre-eminently logical opposition. Also it is complete; the other forms are more or less incomplete.

2d. *Contrary opposition* exists between A and E, universal propositions differing in quality only. RULE: **Both cannot be true, but both may be false.** Between A and E there is a *tertium quid*, namely I and O (§ 31). If All S is P be posited, No S is P is sublated, and *vice versa*. But to deny that *All Stars are Planets* does not afford the inference that *No Stars are Planets*, for both are false, since some are, and some are not, I and O.

Contrariety is less logical than metaphysical. Contradiction occurs only in thoughts, and contradictory thoughts cannot coexist. It cannot occur in things, i. e. among real, external objects, contradiction has no place. Contrary thoughts can coexist—indeed always do so—as *white* and *black*, *straight* and *crooked*, *motion up* and *down*. But these cannot coexist in reality, in external things. Hence contradiction is logical, contrariety physical, opposition. Says Aristotle: Body cannot receive contraries; mind can receive contraries; therefore mind is not corporeal.

3d. *Subcontrary opposition* exists between I and

O, particular propositions differing in quality only.
RULE: **Both may be true, but both cannot be
false.** They are compossible. If *Some S is P* be
allowed as true, it may be that *Some S is not P* is
also true. But if I is false, then O must be true,
and *vice versa*. We remark, however, that the
some in the two propositions must be a different
some. If the same *some* is thought, the proposi-
tions are incompossible. Also that if the *some* is
semi-definite, the rule becomes: Both must be true.

4th. *Subalternate opposition* exists between A
and I and between E and O, propositions differing
in quantity only. RULE: **If the universal be
true, the particular is true; if the particular
be false, the universal is false.** This is not
strictly opposition, but rather a specific application
of the Law of Identity (§ 8). No illustration is
needed.

These formal relations arising from opposition
may be tabulated thus:

	Contradictories.	*Contraries.*	*Subalterns.*
Universals	If A is true, O is false,	....E false,	....I true.
	If E is true, I is false,	....A false,	....O true.
	If A is false, O is true.	The others undetermined.	
	If E is false, I is true.		
Particulars	If I is true, E is false.	The others undetermined.	
	If O is true, A is false.		
	If I is false, E is true,	....O true,	....A false.
	If O is false, A is true,	.... I true,	....E false.

Hence by the truth of universals, and by the falsity
of particulars, all others are determined; otherwise
only the contradictory.

Let it be observed that when the proposition is individual (§ 63) all the distinctions in opposition are merged in the simple negative, which is complete contradiction; as, *Caliban is a man*, and *Caliban is not a man*.

§ 84. **Praxis.** Draw an immediate inference from each of the following propositions:

Infer by determination from the three following:

1. War is an evil. (Use *unprovoked*, and *welcomed with ardor*.)
2. The ignorant are ceremonious. (Use *an age*, and *a nation*.)
3. Honesty deserves reward. (Combine this with:) Every man whom we meet is a neighbor.

Infinitate each of the following propositions:

4. Some men's hearts are not in the right place.
5. It is wrong not to reward the deserving.
6. In jewels and gold, men never grow old.
7. There are studies much vaunted, yet of little utility.

Convert each of the following, and affix the symbols as in § 82:

8. None are free who do not govern themselves.
9. With man many things are impossible.
10. Whoso loveth instruction, loveth knowledge.
11. Fair promises are often not to be trusted.

Contrapone and then infinitate the following:

12. Some invisible things are not intangible.
13. Every unjust action is inexpedient.

7

If the following be true, what opposites are true, and what false?

14. By night an atheist half believes a God.
15. No one is always happy.
16. Some democracies are unstable.
17. Some great orators are not statesmen.

If the following be false, what opposites are false, and what true?

18. All self-confident persons have strong will.
19. No honest men become bankrupt.
20. Some private vices are public benefits.
21. Some plants do not produce seed.

II.—THE SYLLOGISM

§ 85. When we are unable to judge directly the relation of two given notions, resort is had to some third notion as a medium, which, being directly compared with each of the former, enables us to see their agreement or disagreement, and consequently to conjoin or disjoin them. This is mediate inference or reasoning (§ 77). For example, take the notion *man*, and the notion *free-willed*. On comparing these, we are unable, perhaps, to judge whether or not this mark belongs to that concept. So we seek a medium of comparison. We find the notion *responsible*, and see directly that *man* involves *responsible*, likewise that *responsible* comprehends *free;* thus we come to see that *man* involves *free*. This is the intensive view. It is formally stated thus:

> Every man is responsible;
> Every responsible is free;
> ∴ Every man is free.

Again, we are unable to judge, perhaps, whether or not *man* is contained under the class *free agent*. But we judge that *man* is contained under the class *responsible agent*, and this under the class *free agent*, and so conclude man to be a *free agent*. This is the extensive view. It is formally stated thus:

> Every responsible agent is a free agent ;
> Every man is a responsible agent ;
> ∴ Every man is a free agent.

A mediate judgment thus formally and fully expressed is called a syllogism. What is subjectively a reasoning is objectively a syllogism. Hence the definition: A syllogism is a reasoning fully and regularly expressed in language. What is meant by *regularly* will more clearly appear hereafter. Another definition is: A syllogism is an inference by which a proposition is derived from two others conjointly, the one being virtually contained in the others.

§ 86. In dissecting the syllogism we find three propositions, two antecedents or premises, and a consequent or conclusion. To conclude is to shut up together in the last proposition notions which stood apart in the first two. The word syllogism also means a collecting together. The following is an example in extension :

All Men are Persons;   =M: — P = Major Premise;
All Slaves are Men;   = S: — M = Minor Premise;
∴All Slaves are Persons; = S: — P = Conclusion.
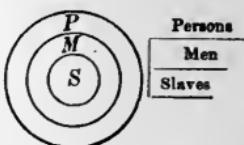
Here are only three notions or terms, *Slaves*, *Men*, *Persons*. These are in the relation of whole and part, *Slaves* being contained under *Men*, and *Men* under *Persons*. *P*, then, is the term of widest extent (as in the notations), or the Major Term; *S* of least extent, or the Minor Term; and *M*, in this

case, of intermediate extent, or the Middle Term. The latter occurs in each of the premises, but not in the conclusion. The two former, called the Extremes, constitute the conclusion. We may now define as follows:

The **Middle Term** (M) is the one with which each of the extremes is compared in the premises. It is also called the Argument, or the Reason.

The **Major Term** (P) is the extreme of greater quantity, or the greater whole. It is always (in extension) the Predicate of the conclusion.

The **Minor Term** (S) is the extreme of less quantity, or the lesser whole. It is always (in extension) the Subject of the conclusion.

The **Major Premise** is the premise containing the Major Term. It is usually placed first. It is also called the Sumption.

The **Minor Premise** is the premise containing the Minor Term. It is usually placed second. It is also called the Subsumption.

Observe that the middle term is not so called because of intermediate extent, but because it is the medium of comparison. There are many cases in which it has not intermediate extent.

The order of the propositions, the major premise first, the minor second, and the conclusion last, is arbitrary, and merely agreed upon for the sake of uniformity. Also that there are three distinct propositions. It is easy and accurate to state the reasoning in an inverted order, and in a single proposition; as, *That slaves are persons is an infer-*

*ence from the judgments that they are men, and that
men are persons.*

In the foregoing example all the propositions are
universal and affirmative. The following differs in
these respects :

No murmurs are prayers...(E)

Some sighs are murmurs...(I)

∴ Some sighs are not prayers.(O)

Prayers.

Murmurs.

Sighs.

$P \longrightarrow : M \longrightarrow , S$
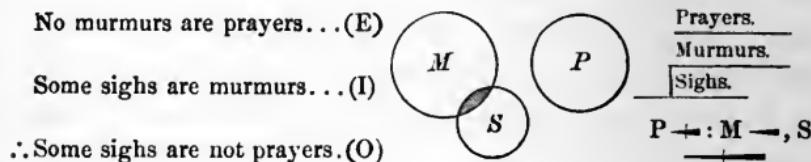
In this example one premise is particular, and one
negative, yielding a conclusion which is both.

§ 87. Let us consider the several notations. The
circular and linear have already been mentioned
(§ 25), and on slight inspection are easily under-
stood. The circular is objectionable, as allowing a
great variety of insignificant arrangement, and also
as constantly signifying too much. For instance,
in the last example it expresses *No S is P*, and
also that *Some S is not M*, or the semi-definite
some. The linear has the advantage that only
those parts of the lines opposite each other are
compared. Of what is beyond the limit of com-
parison nothing is said, the extension of the line
merely serving to show that it is indefinite. For
this and other reasons the linear is preferable to
the circular.

These notations are not at all applicable to in-
tension, but only to extension. But even as ap-
plied to extension they are radically objectionable.

Both circles and lines have geometrical extension only, and are quantities; and therefore when used to figure qualitative notions they transfer thought to the quantitative or mathematical whole (§ 23), and so induce confusion. This is favored by the ambiguity of the word extension, and has doubtless been influential in promoting the unnatural and false view that all propositions are equations, and logic a branch of applied mathematics (§ 74). The circular and linear notations are therefore covertly false and misleading. We shall not, however, wholly discard them, for by long and widely approved usage they have become almost an integral part of elementary logic; but we caution the reader by pointing out their essentially erroneous representation.

Another mode of notation, called graphic notation, is shown in connection with the examples. It needs some explanation. A colon standing next a term indicates that it is distributed; a comma, that it is undistributed. The positive copula is expressed by a pointed dash (–), in manuscript a slight pen-stroke; the negative by the same crossed (⊹). A peculiar advantage of this device is that it discriminately expresses either extension or intension. Pointing to the predicate, the copula indicates an extensive judgment; thus M : – P reads All M is (contained under) P; P ⊹ : M reads No M is (contained under) P; M – , S reads Some S is (contained under) M. The long dash is the copula of the conclusion; thus, P —–— , S reads

Some S is not (contained under) P. Pointing to
the subject, the copula indicates an intensive judg-
ment; thus S : — M reads All S is (comprehends)
M; S , ——·—— P reads Some S is not (does not com-
prehend) P. When needful, a sign of quantifica-
tion (, or :) may be added to the predicate also.
This admirable method of notation is very elastic,
capable of expressing thought relations accurately,
and is recommended for constant use.

§ 88. In the intensive syllogism the predicates
are marks. The following is an example, with its
graphic notation :

$$\text{In Intension} \left\{ \begin{array}{l} \text{Silver is Metallic;} = \text{S} : \text{—M} \\ \text{Metal is Positive;} = \text{M} : \text{—P} \\ \therefore \text{Silver is Positive.} = \text{S} : \text{—P} \end{array} \right\} = \text{S} : \text{—M} : \text{—P}$$

By positive is meant electro-positive. The same
matter transformed yields :

$$\text{In Extension} \left\{ \begin{array}{l} \text{All Metals are Positive elements;} \\ \text{Silver is a Metal;} \\ \therefore \text{Silver is a Positive element.} \end{array} \right\} = \text{P} \text{—} : \text{M} \text{—} : \text{S}$$

Here the relative quantity of the extremes is in-
verted ; the greater part in extension, P, is the less-
er part in intension, and *vice versa* (§ 54). This is
in accord with the law that extension and inten-
sion are in inverse ratio (§ 20). In the example,
Silver comprehends Metallic, and this comprehends
Positive ; S is obviously the greatest whole, and P
the least. Hence, in intension the major term is
the subject of the conclusion, and the minor term

its predicate. And, since it is agreed to place the major premise first, the order of the premises is transposed. Consequently, for changing either form into the other, we have the following RULE: **Transpose the premises, and invert the copulas;** that is, instead of *comprehends*, read *is contained under*, and *vice versa*.

§ 89. The distinction between the extensive and intensive syllogism has been discussed because needful in order to general definition, and to a complete view of the dissected parts. We are now prepared to make an estimate, briefly and once for all, of the value of the distinction. The grammatical difference, which frequently but not always appears, between substantive and adjective noun forms in the predicate is hardly a logical difference. This apart, the external difference lies wholly in transposed premises. But the order of the premises being merely conventional, any distinction founded thereon is arbitrary and artificial, not real and natural, and so goes for nothing. The other difference named in the rule is the inversion of the copulas. This is not an external difference. Ordinarily the copula is wholly indifferent and ambiguous, and its special meaning is indicated only by unusual substitutions (§ 54).

The difference, then, lies entirely in the thought, in the modes extension and intension, and the consequently reversed relation of part and whole. That this is a difference in kind may be granted, one to

be noted in an exposition of mental modes, and in a theory of thought. But it is of very small logical consequence. Both forms of the syllogism are mediate inferences through the same medium ; both reach the same conclusion ; the formal expression of both is the same ; the supreme canon (§ 93) is essentially the same for both ; the general rules (§ 94) are the same ; and the special rules (§ 97) need for adaptation only the interchange of the words *major* and *minor;* hence no general modification of the old logical doctrine is called for by introducing the intensive syllogism.

The practical difference is of no moment. When we consider that one of these modes subjectively, and with the greatest facility, changes to the other, and that without further consequence, we ask: What is the worth of a difference between forms so completely and readily transmutable? The two always actually coexist in thought as psychological correlatives, one more obscure than the other (§ 20), and their convertibility would indicate rather identity, being inconsistent with the opposition which belongs to kinds. Moreover, we very often use both forms in one reasoning. For example:

> All of the metals are positive;......Intensive.
> Silver is one of the metals;........Extensive.
> ∴ Silver is positive.................Intensive.

This is formally perfect, calling for no logical modification whatever.

From these considerations we conclude that it is

needless to continue to observe the distinction. Let the reader, then, understand that hereafter we shall view all matter primarily in the form of extension, even when adjective predicates are used, satisfied that at any instant the view can readily be reversed, and noting expressly the form of intension only in special cases.

§ 90. Formally stated, a syllogism consists of three propositions. But let it not be understood that a syllogistic judgment or reasoning consists of three judgments. Two judgments are premised; then parts of these are combined. This last alone is the act of mediate comparison, the syllogistic judgment, the reasoning; and it is a single act of mind, a single thought, only one judgment. Subsequently it is formally stated as a third proposition, the conclusion.

In the definition of *inference* it is said that something distinct from what is laid down follows of necessity (§ 77). Accordingly, the essence of the syllogism is the necessary sequence of the conclusion from the premises. This necessity flows from the necessary character of the Primary Laws (§§ 5, 7), to which the syllogism conforms, and by which alone it is ultimately governed. It is sometimes expressed in the conclusion by the addition of *must*. For example:

> If all metals are fusible,
> And gold is a metal,
> Gold must be fusible.

The common distinction, then, between demonstrative and moral or probable reasoning lies wholly in the matter, not at all in the form. The form is in all cases demonstrative, apodeictic, necessary.

§ 91. Since logic is not at all concerned with the matter of thought (§ 50), it has no regard for the material truth or falsity of syllogistic propositions, but only for the relation of sequence. In view of this fact, it would be an improvement if the premises in logical examples of the syllogism were expressed not categorically, but conditionally, as in the example in the preceding section. The mind of the reader would then be less drawn to the truth or falsity of the propositions, which is not at all in question, and away from the form, which is the sole consideration. For the same reason, illustrations whose matter is trite and familiar are to be preferred.

But some remark upon the relations of the parts of the syllogism with reference to formal truth and falsity is desirable. The antecedents being granted, the consequent must also be allowed. If the antecedents be true, necessarily the consequent is true. Whatever measure of doubt attaches to the antecedents, just that degree of uncertainty—no more, no less—belongs to the consequent. Should the antecedents be false, it does not follow that the consequent is false; it is merely unproven, and may, perhaps, be established by other antecedents.

These antecedents, being false, prove nothing:

> The natives of Italy were Greeks;
> The Athenians were natives of Italy;
> ∴ The Athenians were Greeks.

This example shows also that the truth of the consequent does not guarantee that of the antecedents. But if the consequent be false, it follows that at least one of the antecedents is false. These points may be summarized thus:

Affirming the antecedents, affirms the consequent.

Denying an antecedent, nothing follows.

Affirming the consequent, nothing follows.

Denying the consequent, denies an antecedent.

§ 92. **Praxis.** Point out the major, minor, and middle terms in the following reasonings, and redress them in syllogistic order. Then write the circular, linear, and graphic notation of each:

1. True poets are men of genius; but since very unwise men sometimes prove true poets, they must be men of genius.
2. Whatever is universally believed must be true. This may be said of the existence of God, which, therefore, must be a truth.
3. No duty involves loss; hence to give freely does not always involve loss, for this is occasionally a duty.
4. Sensualists are not free; for they are governed by passion, and no one so governed is free.

Write the graphic notation of each of the fol-

lowing syllogisms, then change the extensive to the intensive form, and *vice versa*, and write the graphic notation of the result:

5. All men are liable to err;
   None liable to err are safe from disaster;
   ∴ No man is safe from disaster.

6. All expedient actions are justifiable actions;
   Some wars are expedient actions;
   ∴ Some wars are justifiable actions.

Answer the questions appended to the following syllogism:

7. If infants have no language, and if they reason, then some reasoning is possible without language.

   But the sumption is quite doubtful; therefore, what follows?

   But the subsumption is not true; therefore, what follows?

   But the conclusion is not true; therefore, what follows?

   But the conclusion surely is true; therefore, what follows?

   But both of the premises are true; therefore, what follows?

§ 93. The syllogistic judgment that the antecedents necessitate the consequent (§ 90) is determined by the Primary Laws. Since these, however, because of their wide generality, are not readily applicable, the principle of the syllogism is expressed in a single special CANON which can be used as a direct test of its validity. We select four out of many modes of statement.

1. **Part of a part is part of the whole.** As marks are parts of a notion, and species parts of a genus, this is obviously applicable to both extension and intension in the qualitative whole. Also, it applies to the quantitative whole. Its generality, brevity, and simplicity render it very useful. It is, however, inadequate, being applicable only to affirmative syllogisms. A modified formula, limited to the qualitative whole, is: *What is said distributively of a whole may be said of a part.*

Let the reader apply these formulas to any of the foregoing affirmative syllogisms, and the meaning will become clear.

2. *Quicquid de omni valet, valet etiam de quibusdam et singulis. Quicquid de nullo valet, nec de quibusdam nec de singulis valet.* These are the

famous *Dicta de omni et nullo* of the schoolmen.
They have been often and sharply criticised as
senseless, the first being charged with saying
merely *Whatever is true of each, is true of each;* the
second, *What is not true of any, is not true of any.*

3. **Whatever is discovered or admitted as
predicable distributively of a class, must be
allowed as predicable of any of its discovered
or admitted members.**

This formula, carefully worded to avoid similar
reproach, is applicable to syllogisms both positive
and negative, but is limited in expression to those
in extension. We note : *predicable*, positively or
negatively; *discovered*, by intuition, by induction,
or by testimony ; *admitted*, by hypothesis, or mere-
ly for sake of argument ; *of a class*, as an undivided
whole ; *must*, necessity ; *members*, species or indi-
viduals.

4. **Any notion may be replaced by its equiv-
alent; or by its undistributed genus; or, if
distributed, by any of its parts.**

This canon of replacement is here proposed as
more general than the others, and as more truly
expressive of the actual process of thought. It is
simple and self-evident. The first clause is appli-
cable to coextensive, or equipollent, or mathemati-
cal equivalence. For instance :

A is equal to B ;
B is equal to C ;
∴ A is equal to C. .(replacing the first B by its equivalent C).

The several clauses are applicable to the various

forms of immediate inference. For instance, conversion *per accidens* may be viewed thus:

All men are men ;................(mere identity).
Prop..All men are mortals ;.....(*mortals*, genus of men).
∴Some mortals are men..(replacing the first *all men*
    by its undistributed genus).

The view taken in this canon of the qualitative syllogism is peculiar. It considers the sumption as stating a relation between two notions; the subsumption as stating that some other notion is a part of one of them; the syllogistic judgment as replacing that one by this part; and the conclusion as setting forth the result. For instance:

All men are mortal ;.....................(sumption).
Socrates is a man ;.......(he is one, a part of *all men*).
∴Socrates is mortal .....(replacing *all men* by this part).

The third clause of the canon applies thus to syllogisms of the first and second figure; the second clause, to those of the third figure. Moreover, some natural and very simple forms of reasoning, which it is difficult to put in strict logical form, are directly justified by this canon. For instance, the following has been accounted a sore logical puzzle:

The divine law commands us to honor kings ;
Louis XIV. is a king ;
∴The divine law commands us to honor Louis XIV.

Its solution by replacement is easy. In the sumption *kings* is a distributed notion, and in the conclusion is simply replaced by its part, *Louis XIV.* This seems to be the actual mental process by which a child would accept this conclusion.

8.

§ 94. The canon in its original and usual form is directly applicable only to syllogisms of the first figure. For this reason, and because its use as a test is, in some cases, rather confusing, logicians have resolved its principle into a series of distinct GENERAL RULES applicable to any figure. All sound reasoning must conform to these rules. Being quite simple and used separately, they render the process of testing a syllogism easy, quick, and sure. They are as follows:

1. **A syllogism has three, and only three, terms.** For a reasoning, which it expresses, comprises three, and only three, notions—two compared by means of a third. A good syllogism is a tripod. The following is a quadruped; verbally it is a triad, but in thought *quaternio terminorum,* and hence called a quaternion:

> Light is contrary to darkness ;
> Feathers are light ; ....(*light* equivocal).
> ∴ Feathers are contrary to darkness.

2. **A syllogism has three, and only three, propositions.** For three terms give three pairs, and three only. Apparently we have more in—

> All beings that have nerves are sentient.............A
> All self-moving things have nerves.................A
> Worms are self-moving...........................A
> ∴ Worms are sentient ....A

The reasoning is good, the form logical; but we shall find in a subsequent analytical study (§ 106) that it is a Sorites, resolving into two syllogisms of three propositions each.

3. One premise at least must be affirmative.
For if the middle term agrees with neither of the
other two, we cannot infer through it whether or
not they agree with each other. From the follow-
ing negative premises—

No marble is sentient ....E
Some statues are not marble ....O

we get no conclusion; for, however true it may
be, they do not prove some statues not sentient.
But the following yield a conclusion:

No man is without religious feeling................E
Many men are not true believers I
∴ Many infidels are not without religious feeling......O

But the minor premise is really affirmative, the
negative particle belonging to the predicate, which
thereby becomes equivalent to *infidels*, and consti-
tutes the subject of the conclusion.

**4. If one premise be negative, the conclu-
sion must be negative.** For if one extreme be
denied to the middle term, it must be denied final-
ly to the other extreme which agrees with the
middle term by Rule 3. For example:

Few men weep ....O
All men feel ....A

We cannot conclude *Some who feel weep.* How-
ever true it may be, these premises do not yield it.
Few is essentially negative (§ 65), and gives a nega-
tive sumption, yielding a negative conclusion:

Sumption,........Most men do not weep...........O
Now subsume, ...All men feel....................A
Hence conclude,..Many who feel do not weep......O

5. **The middle term must be total at least once.** For if in each premise it is used in a partial sense, it may in each denote different objects, and so be equivalent to two terms, making four in all, in violation of Rule 1. From these premises—

> Some of our students use profane language......I
> Some of our students are refined gentlemen......I

we can conclude nothing, for the middle evidently refers to entirely different groups. This is the fallacy of *undistributed middle.* Sometimes it is not quite so obvious ; for example:

> A valid syllogism has three terms..............A
> This syllogism has three terms................A
> ∴ This is a valid syllogism......................A

Here the middle is in each case the predicate of an affirmative, and is not distributed (§ 75), and so the conclusion is unproven.

If, however, an undistributed middle be so quantified that the sum of its portions is more than the whole, a conclusion is competent. This is called the Ultra-total Quantification of the Middle Term—

> Two thirds of mankind are Asiatics......I
> Two thirds of mankind are heathen......I
> ∴ Some heathen are Asiatics..............I
> (At least one half are, perhaps all are.)
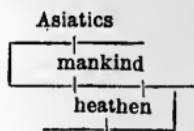
The early logic makes no mention of this apparent exception to the rule, which is apparent only, not real, for the reasoning is in the quantitative, rather than in the qualitative, whole.

6. **An extreme, if partial in a premise, must be so in the conclusion.** For if only *some*

is premised, we cannot conclude *all;* we cannot argue from part to whole. The violation of this rule is the fallacy of *illicit process* (§ 79). It is illicit major or illicit minor, according to the term to which the fault attaches. For example:

All birds are winged........ ....A
A bat is not a bird...........................E
∴ A bat is not winged...........................E

Here the major term *winged* is not distributed (i. e. is partial) in the premise, since it is there the predicate of an affirmation; but it is distributed (i. e. is universal) in the conclusion, since it is there the predicate of a negation. Hence there is an illicit process of the major term. The following is an illicit process of the minor term:

Persons without imagination are not true poets..E
Good logicians are often without imagination I
∴ Good logicians are not true poets..............E

Illicit major occurs only when the conclusion is negative. Illicit minor occurs only when the conclusion is universal.

7. **One premise at least must be universal.** For if the premises be I I, there is no distributed term for a middle (Rule 5). If they be O O, both premises are negative (Rule 3). If they be I O or O I, there is but one term distributed, the predicate of O; if this be taken for the middle term, then illicit major, since the negative conclusion required by Rule 4 distributes its predicate, the major term; if it be not so taken, then undistributed middle (Rule 5).

8. **If one premise be particular, the conclusion must be so.** For a universal following

A with I would require 2 distributed terms ; there is but 1 ;
A " O   "   "   3   "   "   "   are but 2 ;
E " I   "   "   3   "   "   " . " " 2 ;
E " O, both negative (Rule 3).   No conclusion whatever.

§ 95. **Praxis.** Reduce all propositions to strict logical form (§ 61), and arrange them in syllogistic order (§ 86).

Apply the first canon, pointing out the parts and whole, to—

1. The truly virtuous are truly happy. The poor are often the one, and therefore the other.

Apply the third canon, pointing out the class and its member, to—

2. All planetary bodies move in elliptic orbits (*by induction*). Now, if an asteroid be truly a planet (*by hypothesis*), then the orbit of an asteroid is elliptic.

Apply the canon of replacement, supplying the conclusion, to—

3. The gospel promises salvation to the faithful; yet many are faithful whom the world condemns.

Affix the symbol to each proposition, and then point out what rule or rules, if any, are violated in the following examples :

4. Many who conquer their passions have strong will; Whoever resists temptation conquers his passions;
∴ Whoever does not yield possesses powerful will.

5. No sentient being is without a nervous system;
   The sensitive mimosa is not sentient;
   ∴ The sensitive mimosa has no nervous system.

6. Whatever causes intoxication should be prohibited;
   The use of wine causes intoxication;
   ∴ The use of wine should be prohibited.

7. No one is rich who is not content;
   No miser is content;
   ∴ No miser is rich.

8. Few men are entirely unworthy of respect;
   Most men are unlearned;
   ∴ Some unlearned men are worthy of respect.

9. Some x is y; every y is not z; hence some x is not z.

10. No rose is without thorns;
    This bouquet is of roses;
    ∴ This bouquet has thorns.

11. All rational beings are accountable for their actions;
    But many that suffer punishment are irrational;
    ∴ Many that suffer punishment are not accountable for
      their actions.

12. Every man has wants;
    All men are rational animals; ... **afa**
    ∴ Every rational animal has wants.

13. All householders pay taxes;
    The voters are those that pay taxes; .. **afa**
    ∴ All householders are voters.

N. B.—With reference to the forms of examples 12
and 13, see § 146 and § 131.

IV.—FIGURE AND MOOD

§ 96. Syllogisms are divided into Figures according to the position of the middle term. In the First Figure, it is the subject of the major premise, and predicate of the minor. In the Second, it is predicate of both. In the Third, it is subject of both. In the Fourth, it is predicate of the major, and subject of the minor. Thus:

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.
M — P	P — M	M — P	P — M
S — M	S — M	M — S	M — S
∴S — P	∴S — P	∴S — P	∴S — P

sub præ.........tum præ præ........tum sub sub........tum præ sub.

This last line is a useful mnemonic, without any other meaning.

The first figure serves especially to establish general propositions. The universal affirmative **A** can be proved only in this figure. It has been sufficiently illustrated in foregoing examples.

The second figure, whose conclusion is always negative, is especially adapted to proving differences, and so clearing obscure thought (§ 21). For example:

> The true apostles were not thieves;..............**ana**
> Judas was a thief;............................**afi**
> ∴ Judas was not a true apostle...................**ana**

The third figure, whose conclusion is always particular, is especially adapted to bringing in examples, and thus proving an exception to some universal statement. For example:

The apostles sought no temporal reward ;...............**ana**
The apostles were zealous in their work ;...............**afi**
∴Some zealous persons did not seek temporal reward.....**ina**

This contradicts, and so disproves, *All zealous persons seek temporal reward.* Only in the third figure can the middle term be individual; for in each of the others the middle term is once at least a predicate, and an individual cannot be predicated (§ 54). For example:

Peter was an inspired man ;.......................**afi**
Peter was unlearned ;............................**afi**
∴Some one unlearned was inspired..................**ifi**

The fourth figure is reserved for subsequent and special examination (§ 102).

§ 97. By deduction from the General Rules of the syllogism (§ 94) we obtain, relative to the several figures, certain SPECIAL RULES, as follows:

Example.	*Special Rules.*
Fig. 1 (*sub præ*).	
No man is perfect.........	Major premise must be universal. (Else undistributed middle.)
Some saints are men.......	Minor premise must be affirmative. (Else illicit major.)
∴Some saints are not perfect.	
Fig. 2 (*præ præ*).	
No perfect-one is a man....	Major premise must be universal. (Else illicit major.)
Some saints are men.......	One premise must be negative. (Else undistributed middle.)
∴Some saints are not perfect.	(Hence the conclusion is always negative, Rule 4.)

Example.	*Special Rules.*

Fig. 3 (*sub sub*).

　No man is perfect.

　Some men are saints.......Minor premise must be affirmative. (Else illicit major.)

∴ Some saints are not perfect.Conclusion must be particular. (Else illicit minor.)

Fig. 4 (*præ sub*).

　No perfect-one is a man....If either premise be negative, major must be universal. (Else illicit major.)

　Some men are saints.......If major premise be affirmative, minor must be universal. (Else undistributed middle.)

∴ Some saints are not perfect.If minor premise be affirmative, conclusion must be particular. (Else illicit minor.)

These rules, and their proof, should be thoroughly examined; but only those of the first figure need be retained in memory. All have reference to extension. To adapt them to the intensive syllogism, it is needful only to change the word *major* to *minor*, and *vice versa*, wherever they occur. The symbolic notation of the example above is the same for each of the four figures; the graphic notation is different for each of the figures; thus—
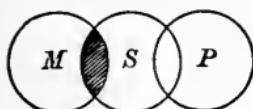
§ 98. The four figures of the syllogism are subdivided into moods, upon the ground of the quantity and quality of the premises. The conclusion need not be taken into account, since it is determined by the premises. A method of ascertaining the moods is as follows:

Relative to quantity and quality, we recognize four propositions, A, E, I, O. These, as premises, taken two at a time, yield sixteen possible combinations, exhibited in the following scheme:

AA	Figs. 1, 3, 4.	EA	Figs. 1, 2, 3, 4.
AE	" 2, 4.	[EE]	3d Gen. Rule.
AI	" 1, 3.	EI	Figs. 1, 2, 3, 4.
AO	Fig. 2.	[EO]	3d Gen. Rule.
IA	Figs. 3, 4.	OA	Fig. 3.
[IE]	6th Gen. Rule.	[OE]	3d Gen. Rule.
[II]	7th " "	[OI]	7th " "
[IO]	7th " "	[OO]	3d " "

But not all these combinations will yield conclusions, for they do not all represent the premises of valid syllogisms. Those bracketed are to be eliminated as violative of the General Rules. Eight (one half) remain as valid, since they accord with the General Rules.

Let us now inquire in which of the four figures each of these eight valid combinations may occur. We apply the Special Rules, and find that EA and EI accord with all these rules, and therefore can appear in each of the four figures, as indicated in the scheme. The figures in which the others can appear are similarly ascertained and indicated. Upon counting, we find there are nineteen valid moods of the syllogism.

§ 99. The first figure has the mood AA. Now annex the symbol of the conclusion, and coin a word containing the three vowels consecutively as the name of the mood, thus: Barbara. The several

moods are treated in this manner, and the names of the nineteen moods thus coined are arranged in the following mnemonic hexameters:

> Barbara, Celarent, Darii, Ferio *que prioris*;
> Cesare, Camestres, Festino, Baroco[1] *secundæ*;
> *Tertia* Darapti, Disamis, Datisi, Felapton,
> Bocardo,[2] Ferison *habet. Quarta insuper addit*
> Bramantip, Camenes, Dimaris, Fesapo, Fresison.

> [2]—or Dokamok,   [1]—or Fakofo.
>   or Fokmafokf.

These names of the moods are very convenient. By applying its name to any reasoning, we at once indicate its figure, and the quantity and quality of each proposition, and also, as will be seen, its relation to other moods to which it may be reduced, and the method of reduction. Moreover, they serve as a test; for, since these are all the valid moods, when we have a simple syllogistic form to which none of the names is applicable, we know at once that the reasoning is false.

From the conclusions it appears that each of the four judgments is proved in Fig. 1. Its four moods are reducible to two, the third and fourth being varieties of the first and second. Thus:

Barbara or *Darii.*	*Celarent* or *Ferio.*
All M is P ;	No M is P ;
All or some S is M ;	All or some S is M ;
∴ All or some S is P.	∴ No S is P,
	or Some S is not P.

Here is one positive and one negative form. Since all the other moods may, as we shall find, be reduced to one or the other of these, they are the two fundamental forms of all reasoning. The evi-

dence of this, furnished by reduction, is perhaps the chief merit of the system.

Again, on noting the conclusions throughout, it appears that—

A is proved in 1 figure and in 1 mood whose initial letter is B.
E   "   3 figures " 4 moods "   "   "   C.
I   "   3 figures " 6 moods "   "   "   D.[1]
O   "   4 figures " 8 moods "   "   "   F.[2]

[1] Except Bramantip.   [2] Except Baroco and Bocardo.

Hence the proposition A is the hardest to establish, and the easiest to overthrow; and O is the easiest to establish, and the hardest to overthrow.

§ 100. Reduction is of two kinds. First, Ostensive Reduction. A syllogism in any other mood may be ostensively reduced to one or another of the first four, and thus brought under the syllogistic canon (§ 93). The initial consonant of each name is that of the mood in Fig. 1, to which it reduces. Baroco and Bocardo are exceptions, but may be replaced by their alternates. The reduction is accomplished by substituting for one or more of the propositions an immediate inference from it. Other consonants in the name direct us in doing this.

    s indicates that the proposition symbolized by the vowel that precedes it is to be converted *simply*.

    p indicates that the preceding proposition is to be converted *per accidens*. (Except in Bramantip, where it shows that, after converting *simply*, a universal is warranted by the premises. This is the reverse of *per accidens*.)

k indicates conversion by *contraposition*.

f indicates *infinitation*.

m indicates that the premises are to be transposed (*mutari*).

The consonants b, d, l, n, r, t, are not significant, but are inserted merely for the sake of euphony, or for metrical quantity.

The following examples will sufficiently illustrate the process:

Fig. 2, *Camestres,*	reduces to	*Fig.* 1, *Celarent.*
All P is M ;		No M is S;
No S is M ;		All P is M;
∴ No S is P.		∴ No P is S.

Cam- Every wicked man is discont'd;   ⎫   ⎧ Ce- No discontented man is happy;
es-   No happy man is discontented;   ⎬ = ⎨ la- Every wicked man is discont'd;
tres. ∴ No happy man is wicked.   ⎭   ⎩ rent. ∴ No wicked man is happy.

Fig. 3, *Darapti,*	reduces to	*Fig.* 1, *Darii.*

Da-   All wits aro dreaded;   ⎫   ⎧ Da- All wits are dreaded;
rap- All wits are admired;   ⎬ = ⎨ ri- Some who are admired are wits;
ti.   ∴Some who are adm'd are dreaded.   ⎭   ⎩ i.   ∴ Some who are adm'd are dreaded.

Fig. 2, *Fakofo,*	reduces to	*Fig.* 1, *Ferio.*

Fak- All murders are intentional;   ⎫   ⎧ Fe- No unintent'l things are murders;
of-   Some homicides are not intent'l;   ⎬ = ⎨ ri- Some homicides are unintent'l;
o.   ∴Some homicides are not murders.   ⎭   ⎩ o.   ∴ Some homicides are not murders.

If in a given syllogism a proposition requiring conversion in order to reduction be an individual proposition, then the reduction is not practicable, for an individual proposition cannot be converted (§ 82).

Moods having the same initial letter conclude the same formal judgment. The only exception is Bramantip, for Baroco and Bocardo have alternates in F.

Moods having the same initial are **equivalent** moods, being generally reducible to each other

by the following GENERAL RULE FOR REDUCTION. **Cause the propositions to appear as required by any legitimate inference from them, transposing, if need be, the premises.**

§ 101. The ostensive reduction just explained could not, it was believed, be applied to the two moods Baroco and Bocardo, having a premise in O. Hence the early logicians devised the *Reductio ad impossibile*. It is a test of the validity of reasoning from granted premises in those two moods.

B, the initial letter, shows, not that the syllogism is reduced to Barbara, but that Barbara is used in making the test.

c indicates that the proposition preceding it is to be omitted, and the *contradictory* of the conclusion substituted. This gives premises in Barbara, from which a new conclusion is drawn. E. g.:

	Baroco,	tested by		*Barbara.*	
Ba-	All murders are intentional;	(1)	Bar-	All murders are intentional;	(4)
roc-	Some homicides are not intent'l;	(2)	ba-	All homicides are murders;	(5)
•	∴Some homicides are not murders.	(3)	ra.	∴All homicides are intent'l	(6)

Here the conclusion drawn in Barbara (6) is false, because it contradicts a granted premise (2). Hence a premise in Barbara is false (§ 91). But one of these (4) having been granted (1), the false one must be the one substituted (5). Now, this false proposition being the contradictory of the original conclusion (3), that conclusion must be true, and this reasoning in Baroco valid. So also:

	Bocardo,	tested by		*Barbara.*	
Boc-	Most men do not weep;	(2)	Bar-	All who feel weep;	(5)
ar-	All men feel;	(1)	ba-	All men feel;	(4)
do.	∴ Many who feel do not weep.	(3)	ra.	∴ All men weep.	(6)

It is sufficient to say : If we contradict the con cluding O, then by plain proof (Barbara) we contradict the premised O, which is absurd, being a self-contradiction.

All the other moods may be tested by the same process. But even in the case of Baroco and Bocardo it is superfluous. The former can be reduced to Fig. 1 by using its alternate Fakofo; the latter by Dokamok (§ 82). But since Bocardo concludes O, it should reduce, not to Darii, but to Ferio. Therefore we propose Fokmafokf as a preferable alternate.

Viewing reduction as a means of testing the validity of syllogisms, then :

Ostensive reduction is direct reduction, and indirect test;
Reduction *ad impossibile* is direct test, and indirect reduction.

§ 102. The fourth figure is open to just and fatal criticism. The general form (§ 96) is :

$$P - M$$
$$M - S$$
$$\therefore S - P$$

But observe that, in the affirmative form, P being the major term, the premises are impossible. The greater cannot be contained under the less. But if S be the major term, we directly conclude $P - S$. This is Fig. 1, t. p. Then we may convert the conclusion *per accidens*, or else simply, and get *Some*

S — P. The procedure is evidently compound, which forbids Bramantip and Dimaris taking rank with the simple moods. In thought they are Barbara and Darii, with transposed premises, which is arbitrary (§ 86), and a subsequently converted conclusion. For similar reasons, Camenes is Celarent.

Fesapo and Fresison are even more faulty. The direct conclusion is illicit major, which is corrected by a conversion *per accidens*. This passage, through a fallacy, is of course inadmissible.

Therefore, the moods of the fourth figure should be rejected as not co-ordinate with the others, as superfluous, and in two cases erroneous.

§ 103. **Praxis.** Write a detailed proof, based on the General Rules, of the three following points:

1. Prove that the conclusion in Fig. 3 must be particular.
2. Prove that the major premise in Fig. 1 must be universal.
3. Prove that from IE no conclusion is valid.

Name the moods expressed by these notations:

4. P — : M —, S
5. P : — M + : S
6. P +, M : — S
7. P, — M : — S
8. P + : M, — S
9. P : — M +, S

In the following examples the agreed order of the propositions is preserved. Redress each in strict syllogistic form, supplying any lacking proposition, and name its mood. Then write its graphic notation. Then, if it be not in Fig. 1, reduce it

9

thereto. To Baroco and Bocardo apply **the test** *per impossibile.*

10. Whoever possesses prudence possesses all virtue ;
    Whoever possesses one virtue must possess pru
    dence.

11. Prudence has for its object the benefit of indi·
    viduals ;
    But prudence is a virtue.

12. No good action results in evil ;
    Some alms-giving results in evil.

13. All abstract studies strengthen the intellect ;
    Exercises that strengthen the intellect are profitable.

14. No science is capable of perfection ;
    All science is worthy of culture.

15. No vicious conduct is praiseworthy ;
    All heroic conduct is praiseworthy.

16. All pride is inconsistent with religion ;
    Some pride is commended by the world.

17. All true philosophers account virtue a good in itself ;
    The Epicureans do not account virtue a good in
    itself.

18. A fallacious argument is not a legitimate mode of
    persuasion ;
    A legitimate mode of persuasion sometimes fails to
    convince ;
    ∴ Not all those arguments are fallacious that fail.

19. Every candid man acknowledges merit in a rival ;
    Every learned man does not do so ;
    ∴ Every learned man is not candid.

20. A few men at least are truly honorable, yet all have imperfections; hence some are so who have imperfections.

21. All expedient acts are conformable to nature;
Nothing conformable to nature is hurtful to society.

22. Nothing that must be repented of is desirable. Now many of our most intense enjoyments constrain repentance. Few of these, then, are truly desirable.

23. There is no growth without sunshine, and these flowers, being deprived of it, will not grow.

24. What is not in Scripture is not binding on conscience;
Since many ecclesiastical canons are not found therein, they may be disregarded.

25. No virtue is a natural quality;
Every natural quality has God for its author.

26. Some kinds of anger are not unrighteous;
Every kind of anger is a passion.

27. Some of our tax-laws are oppressive measures;
All oppressive measures should be repealed.

28. Prejudices are in no case compatible with perfection;
Yet some are quite innocent.

29. All wicked men are discontented;
Socrates is not discontented.

V.—MODIFIED FORMS

§ 104. The various modes in which reasonings may be expressed are endless. Except in treatises on logic, it is rare that a formal syllogism occurs. In conversation, or even in argumentation, its presence is offensive, for an intelligent hearer does not need complete statement, a hint being often sufficient. Unnecessary words do not elucidate, but obscure, thought. It is usual, then, to abbreviate expression. Even essential propositions, if they be obvious, are elided; often they are compounded or condensed, so that the thought is rarely stated entire, or in strictly logical order. We propose now to illustrate some of these modified forms.

An **Enthymeme** is an incomplete syllogism, one or two judgments being unexpressed. There are four orders:

1st. The major premise unexpressed. This occurs most frequently because the sumption is very often a general rule understood and admitted, whereas the subsumption is often a question of fact which needs to be stated and established, in order to be subsumed. E. g., *Yonder celestial body has a proper motion among the fixed stars; therefore it is a member of the solar system.*

2d. The minor premise unexpressed. This gives emphasis to the conclusion. E. g., *Prayers are often sinful; for whatsoever is not of faith is sin.*

3d. The conclusion unexpressed. Sometimes this is high art. The speaker does not formally commit himself, the hearer draws the conclusion, as in the famous speech of Antony over the body of Cæsar. E. g., *Virtue is always discreet; but there is a zeal without discretion.*

4th. Only one judgment expressed. When we see on a tombstone *The memory of the just is blessed*, the implied syllogism is manifest. This form often occurs in texts, proverbs, pithy sayings, and in witticisms. If some one, seeing me vexed, should say, *The way of the transgressor is hard*, I am indignant, for the implied syllogism concludes me a transgressor, and that through an undistributed middle. This was precisely the argument of Job's comforters. Sometimes this form is an insinuation, as when Falstaff replies to Prince Hal, *Lord, Lord, how this world is given to lying!* The answer to a question is often indirect, merely giving a premise which authorizes the doubtful proposition. E. g., *Is smuggling a crime?* Ans., *Whatever violates the rights of society is a crime.* The message to Pilate from his wife may be taken as an instance of a single word hinting premises supporting the hortatory conclusion: *Have thou nothing to do with that JUST man.* Finally, when the disciples of John asked our Lord, *Art thou he that should come?* he replied indirectly, giving them a

minor premise, not in words, but in deeds. In that same hour he did many miracles, and bade the disciples tell John what they had seen.

§ 105. An **Epichirema,** or reason-rendering syllogism, is one that has attached to either premise, or to both, a supporting reason. That is to say, it is a syllogism having for a premise the conclusion of an enthymeme. This enthymeme may, of course, be expanded into a syllogism. A syllogism whose premise is the conclusion of another is called an Episyllogism. One whose conclusion is the premise of another is called a Prosyllogism. For example:

<table>
<tr><td align="center">Episyllogism.</td><td align="center">Prosyllogism.</td></tr>
<tr><td>Vice is odious;</td><td>Whatever enslaves is a vice;</td></tr>
<tr><td>Avarice is a vice; for it enslaves; =</td><td>Avarice enslaves;</td></tr>
<tr><td>∴ Avarice is odious.</td><td>∴ Avarice is a vice.</td></tr>
</table>

The propriety of thus, in the progress of an argument, offering some reason or reasons in support of its doubtful propositions is apparent. By so doing we avoid the necessity of returning over the same ground; and by clearing doubts as we go along, we are not so likely to excite in the hearer the disgust that comes of suspense.

§ 106. A **Sorites** is a chain of enthymemes, holding throughout the relation of prosyllogism to episyllogism. It is expressed either intensively or extensively. The difference between the two forms as to the order of premises is merely conventional, not essential (§ 86).

SCHEME OF SORITES.

Progressive form, in intension.
$$\begin{cases} A \rightarrow B \\ B \rightarrow C \\ C \rightarrow D \\ D \rightarrow E \\ \therefore A \rightarrow E \end{cases}$$

Regressive form, in extension.
$$\begin{cases} D \rightarrow E \\ C \rightarrow D \\ B \rightarrow C \\ A \rightarrow B \\ \therefore A \rightarrow E \end{cases}$$

Resolution of the progressive form.

A is B;	a is c;	a is d;
B is C;	C is D;	D is not E;
∴ a is c.	∴ a is d.	∴ A is not E.

Resolution of the regressive form.

D is not E;	c is not e;	b is not e;
C is D;	B is C;	A is B;
∴ c is not e.	∴ b is not e.	∴ A is not E.

Example.

Some who are prosperous are avaricious;
The avaricious are intent on gain;
The intent on gain are discontented;
The discontented are not happy;
Some who are prosperous are not happy.

Example.

No discontented men are happy men;
All men intent on gain are discont'd men;
All avaricious men are men intent on gain
Some prosperous men are avaricious men;
∴ Some prosperous men are not happy men

Notation in depth.

A, ─ B: ─ C: ─ D: ╅ E
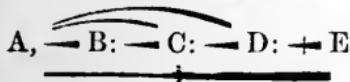
Notation in breadth.

E ╈ :D ─ :C ─ :B ─, A

Other notations in breadth.

E	happy men
D	discontented men
C	men intent on gain
B	avaricious men
A	pros perous men
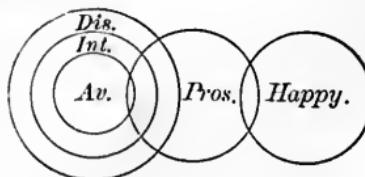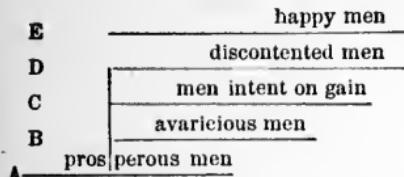
The following points should be carefully noted and analyzed:

1st. The regular Sorites has as many middle terms, and hence resolves into as many syllogisms, as it has premises, less one.

2d. The first proposition is the only major premise expressed; the other premises are minors.

· 3d. Each unexpressed major premise is the conclusion of the preceding syllogism.

4th. Only one premise may be negative, and this must come last in intension, and first in extension ; else illicit process.

5th. Only one premise may be particular, and this must come first in intension, and last in extension ; else undistributed middle.

We also remark that in the scheme all the syllogisms are in Fig. 1. A sorites cannot occur in the other figures throughout. One step, however, may be in Fig. 2 or Fig. 3, but only one, and it must be either the first or the last.

§ 107. Arguments are frequently stated in what at first glance appears to be a single simple syllogism, but which a slight inspection discovers to be compound, or to involve some deviation from rule.

When a conclusion is a compound proposition, it is evident that there must be at least one compound premise, and that the statement involves two or more syllogisms. For example:

> The triumvirs were ambitious;
> Cæsar, Pompey, and Crassus were triumvirs;
> ∴ Cæsar, Pompey, and Crassus were ambitious.

Here are obviously three syllogisms involved in one statement. If we substitute for the major term *founded the empire*, then there is but one, since the change makes all the propositions simple.

When the conclusion is simple, a compound premise involves a surplus of matter. For example:

> Whatever revolves about the earth must present phases,
> The moon alone revolves about the earth;
> ∴ The moon makes phases.

This compound minor premise resolves into *The moon revolves about the earth*, from which the conclusion follows, and *What is not the moon does not revolve about the earth*, from which no conclusion is competent, since it would give illicit major. Hence more is contained in the premises than can be collected in the conclusion.

But a compound exponible premise in other cases may yield a compound conclusion collecting all that is given. For example:

> Justification comes by faith alone;
> Our highest hope is justification;
> ∴ Our highest hope comes by faith alone.

This may be resolved into two simple syllogisms, Barbara and Celarent. But it is not requisite, for we may view *comes by faith alone* as simply the major term, and the whole as Barbara.

There is a class of disguised syllogisms in which the premises are irregularly stated. They consist of simple propositions indeed, but require, in order to bring them under logical rule, the substitution of equipollent propositions, or else of one or more subsidiary inferences. In some cases the resolution is obvious. For example:

> The sun is a thing insensible;
> The Persians worship the sun;
> ∴ The Persians worship a thing insensible.

Here are five terms; yet the reasoning is evident-

ly very good. The canon of replacement is directly applicable, the conclusion being obtained by replacing, in the minor premise, *the sun* by its undistributed genus, *a thing insensible*, as declared in the major premise. But even under the common logical rules the resolution is very simple. From the major premise we may immediately infer, by determination (§ 80), *They who worship the sun worship a thing insensible*, and we then have a perfectly regular Barbara.

The following would hardly puzzle a tyro:

> Whoever probes a wound is on the verge of crime;
> A wound is probed by the healer;
> ∴ The healer is on the verge of crime.

For the passive minor, substitute the equipollent active form (§ 78), *The healer probes a wound*, and we have again Barbara.

An example involving an immediate inference in opposition is as follows:

> That riches are often a bitter curse is true;
> And yet it is also true that most men desire riches;
> ∴ It is false to say that no men desire what is often a bitter curse.

The syllogism which is here slightly disguised is the following Darii:

> They who desire riches desire what is often a bitter curse;
> Most men desire riches;
> ∴ Most men desire what is often a bitter curse.

This major premise is immediately inferred by determination; the conclusion, by opposition; for if E be false, then I is true.

§ 108. There are certain modes of procedure in argument which, though strictly belonging under a doctrine of method, may fairly be mentioned here.

The *argumentum ad rem* is the direct or ostensive proof of the thesis, or problem, or main point in question, the *quæsitum*.

In order to such procedure, premises must be had. To assume them without proof is to beg the question or principle, *petitio principii* (§ 146). If they be granted *argumenti gratia*, or allowed as unquestionable, the procedure is legitimate. But whence come unquestionable premises? To say they are conclusions of precedent inferences is insufficient, for the question recurs as to these. The answer is that ultimately they are derived from pure intuition or from experience, the two original sources of all knowledge.

When the ultimate premises are intuitive principles, self-evident truths, axioms, the procedure is *a priori*. So from the axiom, *Two straight lines cannot enclose an area*, geometry is evolved; from the primary laws of thought, logic; from the moral law, ethics. The syllogistic, deductive process *a priori* is strictly demonstrative, apodeictic (§ 90). And since the ultimate premises are necessary truths, the conclusions are necessarily true.

When the ultimate premises are empirical, or truths of experience, they have been obtained *a posteriori*. Thus induction infers from particular facts of experience truths of empirical universality, and so affords premises for subsequent deduction (§ 77);

such as *All men are mortal*, and *The volume of a gas is in inverse ratio to the pressure.* Hence arise the inductive sciences; as astronomy, geology, physics. Probable or moral reasoning, or dialectics, always involves empirical matter, and so falls short of strict demonstration.

The *argumentum a fortiori*, which may be taken as one variety of that *ad rem*, and understood to mean *for a stronger reason*, gathers up in the conclusion an additional force from relations in the premises. The general formula is: If A be contained under B, and B under C, then *by so much the more* is A contained under C. For example: *If God so clothe the grass of the field, shall he not much more clothe you?*

When unquestionable premises as a basis for direct probation are not available, resort is often had to one of the three following indirect methods:

The *argumentum ad verecundiam* is an appeal to authority, to some venerable institution, to antiquity, etc., as when a dictionary is allowed to settle the disputed meaning of a word, or reference is made to an orthodox creed.

The *argumentum ad judicium* is an appeal to the judgment or common-sense of mankind. We hear it often in conversation in the phrases *Everybody says*, and *No one thinks*, etc.

The *argumentum ad populum* is an appeal to principles cherished by the public. It is legitimate if the principles be sound. But an appeal to prejudice or passion usually betrays weakness.

The *argumentum ad impossibile* or *reductio ad absurdum* indirectly proves a thesis by showing that its contradictory is absurd, that it is self-contradictory, or contradictory of an axiom or other admitted principle, as in § 101. For example, *In a triangle the sides opposite two equal angles are equal; for if they be not equal, it follows that a part is equal to the whole, which is absurd* (§ 10). Likewise it is used in disproof; as, *If the foot-tracks were made by the prisoner, he was wearing shoes much smaller than his feet.*

The *argumentum ad hominem* is arguing from the premises of an opponent merely to defeat him. We accept his principles on which to base a counter-argument, even if believing them false, our argument being directed against him personally, *ad hominem.* It aims to convict him of ignorance, bad-faith, inconsistency, or illogical reasoning, and so to put him *ex curia.* Usually it attempts no more. Our Lord often used this method to silence his adversaries, as in Matt. xxii. 41–45. Since the argument proceeds *ex concesso*, it is formally introduced by a concessive proposition; as, *Though one rose from the dead* (Luke xvii. 31); and, *Though rich, yet not therefore happy, for*, etc. Criticism is mostly in the form *ad hominem*, and should be distinguished from proof of the opposite or controversy.

We remark, finally and generally, that in disproof the attack may be directly on the thesis, showing

it to be false, or upon the argument, showing it to be from a false premise, or else illogical. In the two latter cases the result is merely negative (§ 91), but is often sufficient. The *onus probandi*, or burden of proof, rests ordinarily upon the party making a primary assertion, whether positive or negative. If, however, he can fairly appeal *ad verecundiam* or *ad judicium*, or even *ad populum*, the logical presumption is in his favor, and the *onus* falls on the disputant.

§ 109. **Praxis.** State of each of the following examples whether it is a simple enthymeme, or an epichirema, or a sorites. Put it in strict logical form, and write out the implied syllogisms, naming the mood. In case of an epichirema, distinguish the pro- and epi-syllogism:

1. Blessed are the merciful; for they shall obtain mercy.
2. Cunning cannot be a virtue; for no virtue degrades.
3. Every man should be moderate; for excess will cause disease.
4. Kings, having no equals, have no friends.
5. Suppose ye that these Galileans were sinners above all the Galileans, because they suffered such things? I tell you nay.
6. The flesh of ruminants is good for food, and these animals, since they have horns and cloven hoofs, belong to that class.
7. What if a rule never is, and a principle always is, a law admitting no exception?

8. Whatever tends to withdraw the mind from pursuits of a low nature deserves to be promoted. This classical learning does, since it cultivates a taste for intellectual enjoyments.

9. The Scripture narratives are trustworthy; because the writers had the means of knowing the facts; because they evidently were sincere and candid; and because the narratives are consistent.

10. All true patriots are friends to religion, religion being the basis of national prosperity; but, since their lives are not in accordance with its precepts, it follows that some great statesmen are not friends to religion.

11. Lithium is an element; for it produces an alkali, therefore is a metal, and hence an element.

12. I will not do this act, because it is unjust; I know that it is unjust, because my conscience tells me so; and my conscience tells me so, because the act is wrong.

Put the following logical climax in its opposite form, and write the circular, linear, and graphic notation:

13. The prudent are temperate;
 The temperate are constant;
  The constant are unperturbed;
   The unperturbed are without sorrow;
    Those without sorrow are happy;
     ∴ The prudent are happy.

         —*Seneca, Epist.* 85.

Put the following in its opposite form, and write the notation:

14. Nothing which is indissoluble is mortal ;
     What has no composition of parts is indissoluble ;
       A spirit has no composition of parts ;
         A thinking substance is a spirit ;
           The mind is a thinking substance ;
             ∴ The mind is not mortal.

                    —*Plato, Phædo,* 78.

State each of the following as a regular sorites in either form :

15. A demagogue must hold the people in contempt; **for,** being a favorite, he must know how to manage them ; therefore he understands their weaknesses, and his contempt must follow.

16. We must increase the income-tax; for war has become a necessity, and we cannot go to war without money, which can be raised only by taxation. But the only tax which the resources of the country can bear is the income-tax, since it will fall on the richer part of the population.

Write out the syllogisms involved in the following irregular and compound forms, supplying any inference that may be lacking :

17. The French once more are endeavoring to establish a republic.
     A republic is a representative government ;
   ∴ The French once more are endeavoring to establish a representative government.

18. The value of money is merely a purchasing power ;
     Interest on money is only a reward of abstinence ;
   ∴ Interest on money is not the value of money.

19. Gladstone, Argyll, and Disraeli are eminent states-
    men; but they are also eminent authors;

∴ In some cases literary success is not inconsistent
    with statesmanship.

20. They are out of the reach of their enemies who can-
    not be robbed of what they love;

    He cannot be robbed of what he loves who loves
    God alone;

∴ They who love God alone are out of the reach of
    their enemies.

21. None are happy but the virtuous;

    There are many rich men who are not virtuous;

∴ There are rich men who are not happy.

22. Every one desires happiness; but virtue (alone) is
    happiness; hence every one desires virtue.

23. The true philosopher places his chief happiness in
    moral and intellectual excellence.

    But it is false to say that there is an excellence
    without activity;

∴ His chief happiness is placed by the philosopher
    in moral and intellectual activity.

What names mark the following reasonings:

24. If any objection that can be urged would justify a
    change in the established laws, no laws could
    reasonably be maintained.

25. That used in Luke v. 21; and its answer.

26. That used in Luke xiii. 15–16; and in John x. 34–36.

27. Those used by Demetrius in Acts xix. 23–27; and
    by the town-clerk in vers. 34–41.

28. Those used by the barbarians in Acts xxviii. 3–6.

29. Those used by Paul in Romans v. 7–10.

30. That used by Eliphaz in Job iv. 17–19.

10

VI.—CONDITIONAL PROPOSITIONS

§ 110. The word condition is used in at least three several and important senses, as follows:

1st. A real condition is what must be, that something else may be. Here *must* indicates *conditio sine qua non*, or *necessitas antecedentis*. E. g., *If space is, body may be;* or, more fully, *Space must be, in order that body may be.* So also, *Freedom must be, that responsibility may be.* This primary meaning has reference to reality in objects, and therefore is metaphysical rather than logical.

2d. A causal condition is what determines an event. It is *causa essendi*, an efficient cause of being—*necessitas consequentis*. E. g., *If force is, a change is; If industry is, prosperity is.* In many specific cases the condition, because of an apparent plurality of causes, is not essential or *sine qua non*. Likewise an occasion may be a condition. E. g., *If repentance is, forgiveness may be; If peace* (a negative) *is, prosperity may be.* Deductive logic is not at all concerned with either real or causal conditionals.

3d. A logical condition is what supports a cognition. It is *causa cognoscendi*, an efficient cause of knowing, a reason — *necessitas consequentiæ*.

Very often a real or a causal condition or an occasion is thought merely as a logical condition. E. g., *If space is*, then (I know that) *body may be; If industry is*, then (I know that) *prosperity is; If repentance is*, (I know that) *forgiveness may be.* The inverted real and the inverted causal propositions furnish logical conditions. E. g., *If body is*, (I know) *space must be; If responsibility is*, then *freedom must be; If prosperity is*, surely there is *industry.* But very often in conditional propositions the logical relation of containing and contained, or a reason supporting a conclusion, alone is found. E. g., *If men are, rational beings are; If gold is, metal is; If —— be a man, he is mortal.* In simple forms the logical condition is never *sine qua non.* In compound forms it so occurs.

In its treatment of conditionals, deductive logic is concerned exclusively with the logical condition, with propositions expressing the relation of reason and consequent.

§ 111. The distinction between categorical and conditional propositions has already been noted (§ 58). The general distribution is as follows:

$$\text{Propositions} \begin{cases} \text{Categorical} \dots \text{S is P, and S is not P.} \\ \text{Conditional} \begin{cases} \text{Conjunctive; If A be B, C is D.} \\ \text{Disjunctive; C is either D or non-D.} \\ \text{Dilemmatic; If A be B, C is either D or non-D.} \end{cases} \end{cases}$$

Hypothetical is synonymous with conditional, and hypothesis with supposition. The dilemmatic proposition, because of its compound character, is also

called the conjunctivo-disjunctive proposition. Conditional propositions are always affirmative.

§ 112. A conjunctive proposition expresses the relation between a reason and its consequent. It has two clauses, or members. The subordinate clause expresses the condition, and is called the hypothesis, the supposition, the protasis, the antecedent, or the reason. The principal clause expresses what is conditioned, and is called the apodosis, or the consequent. Usually and formally the protasis is written first, but inversions often occur.

Existential conjunctives have but two terms; formula, If A is, B is; examples in § 110. Another class having but two terms will be noticed presently (§ 116). Conjunctives involving three and four terms are formulated thus:

1 (a)—If A be B, A is C; e. g., If man be responsible, he is free.
  (b)—If A be B, C is A; e. g., If bliss have no anxieties, ignorance is not bliss.
  (c)—If A be B, B is C; e. g., If rubies be clay, some clay is precious.
  (d)—If A be B, C is B; e. g., If metals be fusible, gold is fusible.
2 ——If A be B, C is D; e. g., If the wise be virtuous, Socrates was innocent.

In each of the first forms there are but three terms, one being common to both members. In the second there are four. In simple sequence, the consequent only or both clauses may be negative and may be particular; but the consequent in 1 (b) must be negative, and in 1 (c) must be particular.

§ 113. A disjunctive proposition expresses the *ℓ* relation between two alternative clauses in which one must be true. The formula is: Either C is D, or C is non-D, usually abbreviated as in § 111. <u>One clause is affirmed on condition that the other be denied</u>. In general, then, the condition lies in the opposition of the clauses. The opposed clauses are called the disjunct members, and their relation the disjunction. An inverted formula is: Either D or non-D is C.

This form of judgment involves the principles, and is subject to the laws of Division (§ 29 sq.) and Opposition (§ 83). It implies the division of an unnamed genus into co-ordinate species, and affirms identity between an object or a class of the genus and one or the other of the species. For example: *Carlo is either a dog or a non-dog;* or, naming the genus, *Carlo, being a brute, is either a dog or a non-dog.* So also, *Cares are either distressing or not,* all under the genus *feelings; Every action is either bond or free; Either now or later will suit me very well.*

Disjunctive judgments, to be strictly logical, must make a complete disjunction; that is, the disjunct members must exhaust the divisum, and be exclusive of each other. For example:

Either all wars are evil, or some wars are not evil.	accused	
Either the prisoner is guilty, or he is not guilty.	guilty	not guilty

The members are contradictories within the divided genus or logical universe (§ 27). The law of con-

tradictory opposites is that one must be true and one false; hence, affirming either denies the other, and denying either affirms the other.

It follows that a disjunctive resolves into four conjunctives, thus:

> If C be D, it is not non-D;
> If C be non-D, it is not D;
> If C be not D, it is non-D;
> If C be not non-D, it is D.

Disjunctive judgments often appear in the form: Either C is D, or M is N. Here the matter of the opposed clauses is entirely distinct, and the opposition is mediate, evolved thus:

> Either Richard III. was a monster, or he was not a monster;
> *But* If he was not a monster, Shakespeare was wrong;
> *Hence,* Either Richard III. was a monster, or Shakespeare was wrong.

The alternative is declared, not between members directly opposed, but between one of these and a consequence of the other.

§ 114. When a division is more than dichotomous (§ 31), we have a series of disparate terms, exhaustive and coexclusive; thus:

C is either D, or E, or F, or..........
Bodies are solid, or liquid, or aeriform.

bodies		
sol.	liq.	aer.

Disparates, in logical treatment, must be reduced to contradictories by grouping them into two opposed members; thus:

> Bodies are either solid or (liquid or aeriform=) fluid.

Less than all the members of a disparate series

will not yield a disjunctive judgment, since they are not exhaustive. We cannot say:

Bodies are either solid or aeriform.
Angles are either acute or obtuse.

angles	
acute	obtuse

Hence it appears that contraries, being any two members of a disparate series (§ 31), cannot as such constitute a disjunctive, for both may be false. We cannot say, *Men are either white or black*, for some are red, and so the statement is neither true nor logical. When formal contraries are affirmed disjunctively, it is an indirect assertion that a *tertium* does not exist, and so they become contradictories; as, *Sheep are either white or black*. Also contraries may be stated disjunctively as mere alternatives; as, *Speak briefly, or be silent*. A copulative proposition, however, is formed from contraries; as, *Ye cannot serve God and Mammon*.

In logical strictness, disjunct members must be not only exhaustive, but coexclusive. Yet we often make an imperfect division wherein the species are communicant or intersect, yielding a specific disjunction wherein both may be true. For example:

Jack is either a fool or a knave.

mischief-maker	
fool	knave

That is, he must be one, he may be both. Hence denying one affirms the other; but affirming one, nothing follows. As this is the principle of subcontrary opposition, we distinguish the form as subcontrary disjunction. The copulative and this

subcontrary proposition are mutually convertible by a sort of contraposition; thus:

Either ye do not serve God, or ye do not serve Mammon....*subcontrary.*
Jack is not both smart and good.......................*copulative.*

Disjunctive judgments always affirm, are always positive, never negative. In cases where denial is possible, it is done by *neither—nor*, thus:

C is neither D nor E.
The boy is neither smart nor good.

character	
smart	good

This, however, is not a disjunctive proposition, but a negative categorical compound, a double denial. In *Give me neither poverty nor riches*, the implied *tertium* is sought. While *either—or* are signs of disjunction, *neither—nor* are not at all disjunctive.

§ 115. A disparate series may be transformed by a supposition, thus:

An angle is either acute, right, or obtuse;
If an angle be not acute, it is either right or obtuse.

Either the doctor is not skilful, or the patient is beyond remedy, or he will recover;
If the doctor be skilful, either the patient is beyond remedy, or he will recover.

Either A is not B, or C is D, or C is non-D;
If A be B, C is either D or non-D.

Such is the genesis of the dilemmatic or conjunctivo-disjunctive proposition, which, as this name indicates, is a compound of the two preceding forms, and hence involves no new principle. It may be defined as a conjunctive having a disjunction in the protasis or in the apodosis, or in

both; or, viewed inversely, as a disjunctive having a conjunction in one or both members.

Usually its forms are said to be numerous and intricate. But we hold:

1st. That a difference in the matter or quality of clauses wherein there is partial identity makes them distinct clauses, having a distinct formula.

2d. That the distinction between contradictory and subcontrary opposition may be disregarded, understanding that each formula represents either.

3d. That trilemmatic, tetralemmatic, and polylemmatic forms are, for logical treatment, to be grouped into dilemmatic contradictory forms.

These points being allowed, the following formulas are exhaustive:

1, **Simple**, (a)—Either if A be B, C is D; or if A be B, E is F;
          —having antecedents identical and consequents disjunct.
    "   (b)—Either if A be B, C is D; or if E be F, C is D;
          —having antecedents disjunct and consequents identical.
2, **Complex**, —Either if A be B, C is D; or if E be F, G is H.
          —having antecedents disjunct and consequents disjunct.

The following are concrete examples of these several forms:

1 (a)—If Socrates was innocent, Anytus was either deceived or perjured.
 (b)—If a man be either well or ill deserving, he is a moral agent.
2 ——If the accused was deliberate, he was criminal; or if not, insane.

§ 116. It is now apparent that the conjunctive judgment is the basis of the conditional forms. Let us, then, inquire more particularly into its significance.

The conjunctive is sometimes thought as a qualified proposition. For example: *If air be pure, it is wholesome.* This taken from testimony, or obtained by induction from experience, does not imply any reasoning, though capable of being construed syllogistically, but is a simple judgment, equipollent with: *Pure air is wholesome.*

More generally, however, a reasoning is implied. Observe that the clauses may be in form either positive or negative, but that in fact they are neither affirmed nor denied. In *If A is, B is*, it is not said that A is, or that B is. In *If virtue be knowledge, it is teachable*, it is not said either that virtue is knowledge, or that it is teachable. The clauses are posited not really, but ideally. Observe also that the proposition as a whole is always and only affirmative. What, then, is affirmed? Merely a relation between the members; not, however, the relation of containing and contained, but a relation of dependence, the relation of sequence. E. g., *If A is, then* (or *it follows that*) *B is.* Or, *B is, if* (or *follows from*) *A is.* Evidently the protasis is a logical condition or reason, a premise, and the apodosis is a consequent or conclusion. Supplying an unexpressed premise, we have:

> (All knowledge is teachable ;)
> If Virtue be knowledge,
> *then* Virtue is teachable..........Barbara.

The conjunctive proposition, therefore, is an enthymeme (§ 104). Since its matter is ideally stated, it affirms a sequence only; it is a judgment con-

cerning judgments, expressing in the purest manner the syllogistic judgment (§ 90).

All the various forms of inference are implied by conjunctive propositions. Immediate inference by determination (§ 80) is affirmed in *If coal be fuel, then cheap coal is cheap fuel;* conversion *per accidens*, in *If triangles be figures, then some figures are triangles*, etc. The latter exemplifies a conjunctive form of two terms only.

Mediate inference is implied by those of three or more terms. Of the forms given in § 112, viewing them as ideal enthymemes and supplying the unexpressed premises, 1 (a) yields Barbara, or other moods of Fig. 1; 1 (b) yields Cesare; 1 (c) yields Darapti; but 1 (d) yields Barbara, confirming the rejection of Fig. 4 (§ 102). The form 2, of four terms, yields a sorites, thus:

> (Socrates was wise;)
> If the wise be virtuous,
> (And the virtuous be innocent,)
> *then* Socrates was innocent.

§ 117. **Praxis.** Name each of the following examples in terms of second intention; designate the section and paragraph it particularly illustrates, explaining how; and then reply to the special points required.

Distinguish the four following, and redress as logical conditions:

1. We may enter, if there be room.
2. If the moon has passed the meridian, it will soon be high tide.

3. If the moon has no atmosphere, it has no twilight.

4. If he happened to be there, you surely met him.

Are the following seven examples categorical or conditional?

5. I will not let thee go, unless thou bless me.

6. Until the night comes, we must work.

7. Is any among you afflicted? let him pray.

8. Lear is either at the hut, or at the palace.

9. Hiawatha left his hut or wigwam.

10. It has not been decided whether the war will continue or not.

11. Neither flattery nor threats could prevail.

Having described the following thirteen examples, reduce disparates to contradictories, also subcontraries to copulatives, and *vice versa:*

12. They who slew Cæsar are either patriots or parricides.

13. Either Cæsar was ambitious, or Brutus was criminal.

14. Either if this be a judgment, it affirms or denies; or if it be a question, it does neither.

15. The sun moves round the earth, or the earth moves round the sun.

16. A woman either loves or hates; she never thirds it.

17. Punishment is intended either to repress crime or to reform the criminal.

18. Your god either is talking, or is pursuing, or is in a journey, or peradventure he sleepeth. (*From this obtain a conjunctivo-disjunctive.*)

19. Wherever there is smoke, there is fire.

20. Whenever the moon is on the ecliptic, there is an eclipse.

21. There could be no choice, were there no difference.

22. Day and night are never simultaneous.

23. Every man is already either justified or condemned. (*What genus is here divided?   Reduce to conjunctives.*)

24. Either my wish is fulfilled, or you have disappointed me. (*Mediate.   Evolve, as in* § 113.)

Having designated the names and specific forms of the following examples, reduce the conjunctivo-disjunctives to disparate disjunctives, and *vice versa:*

25. If Cæsar live, he will either rule or ruin.

26. If we go to war, we must either contract a debt, or increase taxation, or indemnify ourselves at the enemy's expense.

27. If my chess-king be moved, or if he be covered, or if I capture the attacking piece, nevertheless I shall be checkmated at the next move.

28. Either if education be popular, compulsion is unnecessary ; or if it be unpopular, compulsion will be resisted ; or if the people be indifferent, compulsion will be fruitless.

29. The mastery of an abstruse science, unless there be competent instruction, is hardly possible, or at best is imperfect.

Complete in syllogistic form the reasonings implied in the first four examples in § 112.

Why must the consequent in 1 (b) be negative?

Why must the consequent in 1 (c) be particular?

VII.—CONDITIONAL SYLLOGISMS

§ 118. The various forms of the conditional proposition are used, without regard to their implied reasoning, as premises in further reasoning. A few illustrations shall suffice.

The following is Barbara, easily solved by replacement (§ 93):

If the using of credit be a demand for goods, all forms of credit affect prices;
But bills of exchange are a form of credit;
∴ If the using of credit be a demand for goods, bills of exchange affect prices.

The following is Camestres, from a disjunctive premise:

All sciences are either pure, inductive, or mixed;
Astrology is neither;
∴ Astrology is not a science.

The following, from a conjunctivo-disjunctive, is Barbara with transposed premises:

If a ruler make an entirely unselfish use of despotic power, he must be either a saint or a philosopher;
But saints and philosophers are rare;
∴ Those rulers who so conduct themselves are rare.

The following is a sorites formed of conjunctives, and resolving into two syllogisms:

> If the Scriptures be the word of God, they should be clearly explained;
> If they should be clearly explained, they must be diligently studied;
> If they must be diligently studied, an order of men must be devoted to them;
> ∴ If the Scriptures be the word of God, an order of men must be devoted to them.

The foregoing are strictly and properly conditional syllogisms, though this title has been usurped by other forms (§ 119 sq.). They may be distinguished from categorical syllogisms, but evidently the difference is not essential (§ 58).

§ 119. Early logicians devised a system of conditional forms, using the terminology of the syllogistic forms. Of these there are four kinds.

The so-called conjunctive syllogism has for a major premise a conjunctive proposition, the minor premise and conclusion being the assertion or denial of the component clauses. It is governed by the following axioms:

1. **Asserting the reason asserts the consequent.**

2. **Denying the consequent denies the reason.**

But denying the reason does not deny the consequent, and asserting the consequent does not assert the reason; for the consequent may follow from some other reason (§ 91). If the protasis be in fact a *sine qua non*, it should be expressed by *Only if*, which is a compound form.

The double axiom gives rise to two so-called moods. The forms of the conjunctive syllogism in these moods are as follows:

Modus Ponens (constructive).	If A be B; But A is B; ∴ C is D.	then	C is D; But C is not D; ∴ A is not B.	Modus Tollens (destructive).

If the people are industrious, wealth is increasing;

Ponens.—	They are industrious; ∴ Wealth is increasing;	Wealth is not increasing; —Tollens. ∴ The people are not industrious.

The sumption affirms, though one or both clauses be negative. It alone is conditional, the rest are categorical. There may be four terms, as above; all occur in the sumption. Hence the subsumption has no new term, and the conclusion may have nothing in common with it.

From the axioms two Rules are derived serving to guide and test. They are as follows:

1. In **Ponens** the subsumption and conclusion must each agree with its corresponding clause in both quantity and quality.

2. In **Tollens** the subsumption and conclusion must each disagree with its corresponding clause in both quantity and quality.

Conclusive deviations from these rules will, on inspection, be found to lack logical accuracy. The double disagreement in Tollens is because logical denial is only by contradiction. When the subject is individual, or a generic total, as above, its quantity being fixed, contradiction is merely a change of quality.

The following example illustrates the rules:

	If any nation prosper, all are benefited;	
	Some are prospering;	Some are not benefited;
Ponens.—or	This one prospers;	or That one is not; —Tollens.
	∴ All are benefited.	∴ None are prospering.

Negative clauses, one or both, conform strictly to the rules. Thus:

	If A be not B, then C is not D;	
Ponens, *asserts.*—	A is not B;	C is D;—Tollens, *denies.*
	∴ C is not D.	∴ A is B.

On contraponing the sumption—that is, taking the negative of each clause and then transposing them—we find that the moods are mutually reducible.

In *reductio ad absurdum* it is usual to state the argument hypothetically; then the tollent mood is often so obvious that it is not expressed; e. g., *If we say we have not sinned, we make God a liar.*

§ 120. <u>The disjunctive syllogism has for its major premise a disjunctive proposition whose disjunction is resolved in the minor premise and conclusion.</u> It is governed by the axioms of contradiction and excluded middle (§§ 9, 10). The disjunct members being contradictory, affirming one denies the other, and *vice versa.* This yields two moods, each double, thus:

Modus Tollendo Ponens.	{	C is either D or E (=non-D);		Modus Ponendo Tollens.
		C is not D;	C is D;	
		∴ C is E.	∴ C is not E.	
"		or	or	"
"		C is not E;	C is E;	"
"		∴ C is D.	∴ C is not D.	"

11

Either all men are justified, or some are condemned;

TOLLENDO — Some are not justified; | All are justified;—PONENDO
PONENDS.—∴ Some are condemned. | ∴ None are condemned.—TOLLENS.

or	or
" None are condemned;	Some are condemned; "
" ∴ All are justified.	∴ Some are not justified. "

The sumption always affirms. The conclusion has
the same quantity as the subsumption, but the
opposite quality.

When the disjunction is subcontrary (§ 114), we
may proceed in the ponent moods, but not in the
tollent. For example:

> All afflictions are either punitive, or tentative,
>   or disciplinary;

To. PONENS.—   Job's afflictions were neither punitive nor dis-
>   ciplinary;
>   ∴ They were tentative.

     "   David's were not tentative;
>   ∴ They were either punitive or disciplinary (per-
>   haps both).

Positing one subcontrary does not sublate the
other, for both may be true.

A copulative proposition involving contraries
(§ 114) yields conclusions in the tollent moods, but
not in the ponent. For example:

> Ye cannot serve God and Mammon;

Po. TOLLENS.—   Ye serve Mammon;
>   ∴ Ye do not serve God.

Sublating one contrary does not posit the other,
for both may be false. As the subcontrary dis-
junctive and the copulative propositions contra-
pone into each other, so likewise these syllogistic
forms are mutually convertible.

§ 121. The dilemmatic proposition, being a compound form (§ 115), furnishes a double process. Viewing it as a conjunctive, according to its first definition, and taking the disjunct members as a single clause, we proceed as in § 119; thus:

If A be B, either C is D, or E is F;

PONENS.—But A is B; | Neither C is D, nor   —TOLLENS.
    ∴ Either C is D, or E is F. | E is F;
                         | ∴A is not B.

If the apostles taught falsely, they were either deceivers or deceived.

PONENS. —They did teach falsely; | They were neither de-   —TOLLENS.
    ∴ They were either de- | ceivers nor deceived;
      ceivers or deceived. | ∴ They did not teach falsely.

On the other hand, viewing it as a disjunctive, according to its second definition, we proceed as in § 120; thus:

If A be B, either C is D, or E is F;
To. PONENS.—   But C is not D ;
           ∴If A be B, E is F.

If Socrates was innocent, Anytus was either deceived or perjured;
But Anytus was not deceived;
∴If Socrates was innocent, Anytus was perjured.

By denying *Anytus was perjured*, we have another To. Ponens. The disjunct members being contradictories under the stated condition, yield also two forms in Po. Tollens.

Observe that neither of these forms of the conjunctivo-disjunctive syllogism, though involving each a dilemmatic proposition, treated first conjunctively, then disjunctively, is a dilemma.

§ 122. The dilemma is a conditional syllogism

having a double conjunctive premise and a disjunctive premise. Either may be taken as the sumption, but it is usual to write the double conjunctive first. None of its propositions is dilemmatic. It has three forms, as follows:

1. Simple constructive: If A be B, C is D; and if E be F, C is D;
   PONENS.—   But either A is B, or E is F;
                           ∴ C is D.

2. Complex constructive: If A be B, C is D; and if E be F, G is H;
   PONENS.—   But either A is B, or E is F;
                           ∴ Either C is D, or G is H.

3. Complex destructive: If A be B, C is D; and if E be F, G is H;
   TOLLENS.—   But either C is not D, or G is not H;
                           ∴ Either A is not B, or E is not F.

A single concrete example from Demosthenes de Corona must suffice. It is in the complex constructive form, as follows:

If Æschines joined in the public rejoicings, he is inconsistent;
    if he did not, he is unpatriotic;
But either he did, or he did not;
∴ Either he is inconsistent, or he is unpatriotic.

The form of the sumption in this example may be expressed thus:

If A be B, A is C; and if A be not B, A is D.

Here the first term of each of the clauses is the same, and the antecedents differ only by the negative. Yet the form is complex, for the clauses differ either in matter or in quality (§ 115).

There cannot be both a simple constructive and a simple destructive dilemma. Denying the consequents in No. 1 gives:

If A be B, C is D; and if E be F, C is D;
But C is not D;
∴ A is not B; and E is not F.

This, however, is merely a double conjunctive syllogism in Tollens. The simple destructive form, corresponding to No. 1, is :

> If A be B, C is D; and if A be B, E is F;
> But either C is not D, or E is not F;
> ∴A is not B.

But this is merely No. 1 contraponed, and then treated in Tollens. It is therefore essentially the same, and should not be enumerated apart.

§ 123. Let us briefly inquire into the nature of the forms presented in the three foregoing sections. Are they truly inferences? We recall that deductive inference is of two kinds, mediate and immediate. In mediate inference we determine the relation of two notions through a third, the middle or medium. A syllogism is the formal expression of this mediate process, and hence a middle term is its essential feature. Now, hypothetical or conditional syllogisms, so called, contain no middle term. Therefore they are not syllogisms, not expressive of reasoning at all. Inspect the following :

MODUS
PONENS.
> If law prevails, our rights are
>   secure ;.................*Major Premise.*
> But law does prevail ;......*Minor Premise.*
> ∴Our rights are secure.......*Conclusion.*

There is no term here with which the two terms found in the conclusion are compared in the premises. There are in all four terms, and all found in the so-called major premise. The so-called minor introduces no new matter, and has nothing in common with the conclusion, as in a true syllogism.

Are they immediate inferences? An immediate inference from a given judgment infers directly— that is, without a medium—a different judgment. Let us inspect the same example presented in a slightly varied form :

> If Law prevails, then our rights are secure.
> Law prevails, then our rights are secure.

Now, here is an absolute iteration of thought, stated first as supposititious, then as assertorial. The subject is the same. The predication is the same. The second judgment, then, is not different logically from the first, and therefore this cannot be an immediate inference. In the tollent mood and in the disjunctive syllogism an immediate inference by opposition (§ 83) is indirectly involved.

These forms express primarily the passage of thought from the ideal to the real, from the questionable to the true, some unexpressed ground having been discovered. The process is therefore metaphysical rather than logical (§ 91). Ought not, then, these conditional forms, these pseudo-syllogisms, to be banished from logic ? By no means ; for they are true, natural, and very common modes of expressing thought, and hence call for logical analysis and treatment. Nothing is more common than for a reasoner at the outset to state hypothetically his premise and conclusion. This he does for the sake of clearness, and to show whither he is tending. For example :

> If the prisoner was sane, then he is responsible for his act.

His first argument may be to show the necessity of

the sequence herein declared. As accusing counsel, he next endeavors to establish this antecedent minor, perhaps by showing the deliberation of the agent, his consistency, his motives, etc.; and, it may be, he brings in the testimony of medical experts. When the argument is complete, he closes by declaring categorically:

The prisoner was sane, therefore he is responsible for his act.

Again, many of these conditional forms present exceedingly condensed expressions through which thought darts with rapidity ; and unless the thinker is familiar with their analysis, he is in danger of paralogism, or of being imposed upon by sophism. On the other hand, their condensation gives to a just argument weight, and logical and rhetorical force. They should, then, be discussed, not only as subjects of analysis, but also because of the practical advantages resulting from their close examination.

It is clear, however, that their nomenclature ought to be changed. The unfortunate misapplication of the terms *syllogism*, *major* and *minor premise*, *mood*, etc., and the attempt to enunciate rules and methods of reduction parallel to, but distinct from, those of the true syllogism, have filled logic for centuries with confusion and error. But so deeply rooted in logical literature and so widely spread are this false system and terminology that the needed correction can be made only by the highest authority.

§ 124. **Praxis.** In what moods are the following three syllogistic forms?

1. Every body is solid, liquid, or aeriform;
   Solid, liquid, and aeriform bodies are elastic;
   ∴ Every body is elastic.

2. Memory is either circumstantial or philosophic;
   Also it is either voluntary or spontaneous;
   ∴ In this case, what is either voluntary or spontaneous is also either circumstantial or philosophic.

3. Desires are either spontaneous or voluntary;
   But whatever is voluntary has moral quality;
   ∴ Desires are either spontaneous, or they have moral quality.

Describe each of the examples in terms of second intention; redress in strict form; if inaccurate, say wherein; then reply to specific points.

4. Mohammed was either an enthusiast or an impostor;
   But he was an enthusiast, and therefore not an impostor.—(*Is the disjunction contradictory?*)

5. Unless matter can move itself, its first motion must have been given it by a spiritual being. But matter cannot move itself; therefore, etc.

6. If man cannot make progress towards perfection, we must believe him to be either an incapable brute, or already divine.—(*Ad abs.*)

7. Whether logic be regarded as a means of mental discipline or as a practical guide in reasoning, it ought to be studied. But it is both. Hence—(*what?*)

8. The ancients were in genius either superior to the moderns, or inferior, or equal.—(*How many syllogisms may be based on this?*)

9. If all testimony to miracles is to be admitted, the

mediæval legends are to be believed; but they are
not to be believed, and therefore no testimony to
miracles is to be admitted.

10. There are two things we ought not to fret about:
what we can help, and what we cannot.—(*From
this form a dilemma.*)

11. The greater angle of a triangle is subtended by the
greater side.

If $b > c$, then $B > C$.
For if not, then either $B = C$,
    or $B < C$.
But $B = C$ is not true, for then $b = c$ (I., 5),
    which is against the hypothesis;
Nor is $B < C$ true, for then $b < c$ (I., 18),
    which also is against the hypothesis;
∴ $B > C$.   Q. E. D.—*Euclid, Book* I., *Proposition* 19.
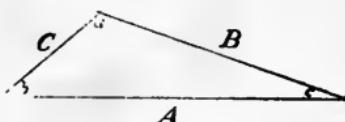
12. If the world existed from eternity, there would be
records prior to the Mosaic; and if it were pro-
duced by chance, it would not bear marks of de-
sign. But there are no records prior to the Mosaic,
and the world does bear marks of design.   ∴ The
world neither existed from eternity, nor is it the
work of chance.

13. A government cannot be at the same time despotic
and the licenser of a free press;
But the English government permits a free press;
∴ The English government is not despotic.

14. If the books in the Alexandrine Library be in con-
formity with the doctrines of the Koran, there is no
need of them; if adverse, then also they should
be burned.

15. If pain be severe, it will be brief; and if it last long,
it will be slight; hence it should be borne patiently.

16. If a man cannot be virtuous, he must be either unable to know what is right, or unable to will what is right. But he is not unable to know what is right, for he is intelligent; nor unable to will what is right, for he is free.

17. We must either gratify our vicious propensities or resist them; the former course will involve us in sin and misery, the latter requires self-denial; therefore we must either fall into sin and misery, or practise self-denial.

VIII.—QUANTITATIVE FORMS

§ 125. The distinction between the qualitative
or logical whole and the quantitative or mathe-
matical whole has already been indicated (§ 23),
and some note made of the latter (§ 24). It is
now needful to examine the quantitative forms of
thought more particularly, because of their essen-
tial difference, and because, though constantly oc-
curring, logicians commonly either neglect them
altogether, or else confound them with the co-ordi-
nate qualitative forms.

In the qualitative whole the thought is funda-
mentally of marks; in the quantitative, of magni-
tudes. A quantity, as distinguished from a quality,
is measurable by some standard or unit of measure,
real or ideal. Magnitudes differ in kind, and when
thought as kinds the notion is qualitative; but
magnitudes of the same kind differ in degree, and
the notion of degree is quantitative, is measurable,
is mathematical. The distinction between kind
and degree is fundamental and thorough-going in
all thinking, and differentiates the two wholes.

§ 126. From its name alone, a common noun, it
is often impossible to decide whether a notion is

qualitative or quantitative. Thus *mankind* in its form is a class, and the *human race* is a mass, an individual, having no species, and can be partitioned only into sections; but *population* may be thought either as a class or as a mass. So *being* or *thing* is a class including all kinds of existences, and the *Universe* is a mass, a mathematical whole, a collection of all things into a unit, the only one not a part of any other, and is capable only of dissection; but, as herein said, *things* may be thought as a collective whole. Again, *animals* may be thought as divisible into kinds, or as the individual sum total of many individuals, severable only, as the part saved in the ark, and the part destroyed by the deluge. The ambiguity of the predesignations *all* and *some* has been noted (§§ 64, 66); hence these do not serve to determine which whole is thought. Generally, if not determined by the context, it is quite ambiguous, the thought readily taking either form, and requiring introspection to ascertain which of the two is thought. So far of general names. Proper names, and common names deprived of their generality by demonstratives, possessives, and the like, designate individuals (§ 63), and the thought is quantitative.

§ 127. Degrees are formally of two sorts, equal and unequal. Hence quantitative judgments or judgments of degree are two, both being mathematical comparisons.

First, in the judgment of equality the ambigu-

ous copula *is* (§ 54) means *is equal to* (=), and when this is so expressed the proposition is unambiguously quantitative. For example: *A is B; The population of London is double that of New York; X=Y*, or *X is equal to Y*, often expressed: *X and Y are equal.*

Second, the judgment of inequality conforms to the axiom, A whole *is greater than* a part, and so has the copula *is a part of*, or its obverse *contains*, or else *is greater than* (>), or its obverse *is less than* (<). When these are expressed, the judgment is unambiguously quantitative. For example: *A is a part of B*, or *B contains A; Maine is a part of New England*, or conversely; or else *A is greater than B*, or *B < A; The earth is greater than the moon*, or conversely. This simple relation is often compounded with other notions; as in *included by, longer* and *shorter, better* and *worse, stronger, more repulsive, most attractive, highest*, etc. Thus degrees of comparison are quantitative. For example: *Men are stronger than boys* means *The strength of men is greater than the strength of boys; Iron is not as heavy as lead* means *The specific gravity of iron is less than that of lead; Lias lies above coal* means *The height* (in the geological scale) *of Lias is greater than that of coal; Women love best* means *The love of women is greater than any other.*

In the qualitative whole an individual cannot become a predicate, and therefore the individual proposition is inconvertible (§§ 54, 82). In the quantitative whole an individual is often the pred-

icate, and all quantitative propositions are always and only simply convertible, the copula in the second class being changed to its obverse.

When the predicate is an individual, or when it is qualified numerically or by some term of measure, or when it is quantified as *all* or *some*, directly or indirectly (§ 74), the proposition is quantitative. E. g., *Aristotle is the father of logic; Thou art the man; This is our home; A legion is* (=) *ten cohorts; His reasons are as two grains of wheat hid in two bushels of chaff; It weighs a pound; All men are all reasoners; Here only thieves* (generic total, not every one) *are to be dreaded*, or *all* (the sum total) *that is to be dreaded; The committee* (collective) *consists of some* (a portion or section) *of our wisest men; The population of London is more than* (>) *a million.* Generally the character of the predicate determines in which whole the proposition lies.

The complete generality of many quantitative forms should be observed. Several of the foregoing examples are cases. Pure mathematics, the science of quantity, treats almost exclusively of such abstract generalities; as $6 = 2 \times 3$; $x^2 - y^2 = (x+y)(x-y)$; *Triangles on the same base, and between the same parallels, are equal.*

§ 128. Inference in the quantitative whole is immediate and mediate. Immediate inference from equivalent propositions conforms to the CANON: **Equals affected by equals are equal.** This is

a general statement of four of the logical axioms (κοιναὶ ἔννοιαι) of Euclid, that if equals be added to, or taken from, or multiplied by, or divided by equals, the results are equal. The process corresponds to Determination (§ 80). E. g., As from *A horse is an animal*, and *What is young is strong*, we may immediately infer that *A young horse is a strong animal*, so from $a=b$, and $c=d$, we may immediately infer that $a+c=b+d$. The principle, in a modified form, applies to propositions of inequality. E. g., *If $a>b$, then $2a>2b$.*

§ 129. Mediate inference from equivalent propositions conforms to the CANON: **Quantities equal to the same thing are equal to each other.** This is Euclid's first logical axiom. The general formula is:

If A = B;	A *is not equal to* B;
and B = C;	Negatively: B *is equal to* C;
then A = C.	∴ A *is not equal to* C.

This may be called the syllogism of equivalence. Obviously it is a specific application of the Primary Law of Identity (§ 8), which is the ultimate principle involved in both the foregoing canons. The first clause of the canon of replacement (§ 93) also justifies the process, and is even more general. A concrete example in this form is as follows:

The density of the human body is the density of water;
The density of water is the density of air taken 817 times;
∴ The density of the human body is 817 times the density of air.

It will be observed that the middle term here is

a standard of measure. And this gives occasion to remark the logical function of standards of measure of all sorts. They furnish the media through which we are enabled to compare quantities which cannot be immediately compared. The yard, the bushel, the pound, the atomic weight of hydrogen, the thermometer, barometer, electrometer, etc., supply us with middle terms through which to reason in our calculations. The metric system furnishes a common middle term, the metre, by which to compare its various standards with each other.

In the syllogism of equivalence the order of premises is obviously indifferent. The order of subject and predicate is also indifferent; that is, either term may be made the subject of thought, and the other the predicate, without other change. The distinction of major and minor terms, and consequently that of major and minor premises, does not exist, the terms being equivalent. The equivalent proposition is always and only simply convertible. The doctrine of Conversion, then, has no place relative to this syllogism. It follows, also, that Figure is of no moment, and is to be disregarded. Moods are reduced to two, the positive and the negative; for the quantification of every term is always total. Hence questions concerning distribution and non-distribution cannot arise.

These eliminations render the logical treatment of this syllogism exceedingly simple. Perhaps from this simplicity it is, as well as from its clear intuitive exactness, that elementary mathematics is with-

in the grasp of immature minds, even children often being able to apprehend it quite thoroughly; whereas reasoning in the logical whole, with the qualitative syllogism as the unit form, requires more mental discipline and maturity.

§ 130. A geometrical example (Euclid, I., 32) conforming to the canon of mediate inference may be stated as follows:

> The three interior angles of a triangle are equal to two right angles;
> For the interior angles are equal to the adjacent exterior and interior angles;
> And these are equal to two right angles.

The expression is rendered more facile by the use of a lettered figure, the letters taking the place of a verbal description of a part; but the processes are identical.

Let us exhibit a slightly varied and redressed proof of the same proposition by aid of a lettered figure; thus:

> Through the apex of an angle b draw a line parallel to the opposite side. Then:
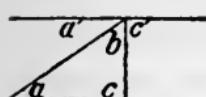
$$a=a'\ldots\ldots\text{Prop. 29.}$$
$$b=b\ldots\ldots\text{Identity.}$$
$$c=c'\ldots\ldots\text{Prop. 29.}$$
$$\underline{a+b+c=a'+b+c'}\text{.Canon of immediate inference.}$$
$$a'+b+c'=2\llcorner\ldots\ldots\text{Prop. 13.}$$
$$\therefore a+b+c=2\llcorner\ldots\ldots\text{Canon of mediate inference.}$$

This last equation may of course be translated into words.

It is needful to remark particularly that whether the proposition be expressed in symbols or in words,

12

both have the same, and indeed a complete, generality. Also that the passing from one to the other is not at all a logical process, but simply a translation of expression. Changing the symbols into words is often spoken of as a generalization or an induction, but it is neither. Nor is the reverse a deduction; yet the following, for instance, is sometimes laid down as a syllogism:
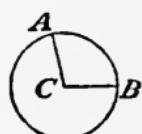
All radii of a circle are equal;
AC and BC are radii of a circle;
∴ AC and BC are equal.

But there is here no progress of thought, no change of thought whatever, only of expression. AC and BC stand for any radii of any circle; hence the simulated conclusion is as completely general as the verbal proposition which simulates a major premise, and nothing whatever is proved.

§ 131. Mediate inference from propositions of inequality conform to Euclid's 9th logical axiom, A whole is greater than a part. This, modified, furnishes the CANON: **A part of a part is part of the whole** (§ 93). Syllogisms in conformity with this canon may be called partitive syllogisms. A single example, and the converse form, shall suffice for illustration:

A minute *is a part of* a degree;
A degree *is a part of* a circle;
∴ A minute *is a part of* a circle.

Converse:

A *contains* B;
B *contains* C;
∴ A *contains* C.

Another modification of the axiom furnishes the CANON: **A greater than a greater is greater still than the thing.** Syllogisms conforming to this canon may be called comparative syllogisms. The formula is:

A > B;		B *is less than* A;	A	————————
B > C;	Conversely:	C *is less than* B;	B	————
∴A > C.		∴C *is less than* A.	C	———

E. g........The planet Jupiter *is greater than* the earth;
The earth *is greater than* the moon;
∴The planet Jupiter *is greater than* the moon.

Logicians sometimes distinguish between the inferences *a minore ad majus* and *a majore ad minus;* but the distinction is superficial, since one is simply convertible into the other.

Observe that the premises authorize a pregnant conclusion, one *a fortiori* (§ 108), usually expressed thus:

∴By so much the more *is* A *greater than* C; or:
∴C *is still less than* A; or:
∴*A fortiori* the planet Jupiter *is greater than* the moon.

The following example is followed by its redressed form:

The tree is higher than the man;
The spire is higher than the tree;
∴The spire is still higher than the man.

The height of the tree *is greater than* the height of the man;
The height of the spire *is greater than* the height of the tree;
∴The height of the spire *is still greater than* that of the man.

The following (Euclid, I., 20, redressed) exhibits a variation in respect of its symbolic statement:

Any two sides of a triangle are greater than the third.

Extend the side A until C′=C.   Then:
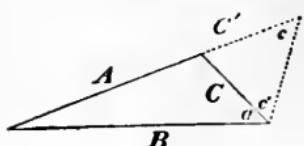
c′=c........Prop. 5.

a + c′>c′.......Ax. 9.

∴ a + c′>c.......Mediate inference.

Then A+C′>B.......Prop. 19.

A+C′=A+C....Ax. 2.

∴ A+C >B.......Mediate inference.

Not only do both kinds of judgments of degree occur in the same reasoning, as in the foregoing demonstration, but qualitative judgments also often combine with quantitative.   For example:

> Regulus is a star of the first magnitude;
> Sirius is as bright or brighter than Regulus;
> ∴Sirius is a star of the first magnitude.

A proposition whose terms are not merely equivalent, but in strict and entire identity (§ 8), that is, in what has been called the sibi-relation, cannot serve as a premise in a proper syllogism; for such terms, differing merely as to words, are one in thought, and consequently we should have a pseudo-syllogism of only two terms, begging the question (§ 146).   Cf. § 26; § 95, Ex. 12; and § 130.

Quantitative relations may be expressed also in the several forms of the so-called conditional syllogism.   For an instance, see § 124, Ex. 11.

§ 132. **Praxis.** State whether the following propositions are qualitative or quantitative.   If the latter, redress with the copula:

1. It is the duty of every man to serve God and honor the king.   Only birds are feathered.

2. George Eliot is Mrs. Lewes.   Arrows are swifter than eagles.

3. God alone is good.   We are all sinners.

4. Every sly act is nothing less than dishonest.

5. The container contains the contained.   That man is my father.

6. None but Aryans are capable of the highest civilization.

Can the deduced formula $Circ. = 2\pi R$, or this *vis viva* $= mV^2$, be generalized?

Name the class to which each of the following reasonings belongs. Supply any lacking proposition. Redress, if need be, exhibiting the copulas. Construe the first as qualitative also:

7. Wisdom is more precious than rubies, and rubies than gold; hence wisdom is of yet higher value than gold.

8. The author of *Athalie* was the greatest French dramatist;
   But Racine was the author of *Athalie*.

9. The market value of my cloke is $15;
   A sword will cost me $10.   (Luke xxii. 36.)

10. John knew more than Peter, and Peter than Mark;
   ∴ John knew more than Mark.

11. Aristotle lived after Plato, and Plato after Socrates;
   ∴ Aristotle lived after Socrates.

12. Virginia is one of the Southern States;
   The Southern States are a part of the Union;
   ∴ Virginia is a part of the Union.

13. Lias lies above Red Sandstone;
   Red Sandstone lies above Coal;
   ∴ Lias lies above Coal.

14. The orbit of Venus is within that of the Earth;
   And this within that of Jupiter;
   ∴ The orbit of Venus is within that of Jupiter.

15. The dome is under the sky, and the altar under the
   dome; therefore the altar is under the sky.

16. Behold, the heaven and heaven of heavens cannot
   contain thee; how much less this house that I
   have builded!

17. It were better to have no opinion of God at all than
   such an opinion as is unworthy of him; for the
   one is unbelief, the other is contumely; and cer-
   tainly superstition is the reproach of the Deity.

Prove the following proposition (Euclid, I., 15),
first in words only, then by the figures and letters,
as in § 130:

18. If two straight lines intersect,
   the vertical angles are equal.

Redress the following demonstration (Euclid, I.,
18) as in § 131:

19. The greater angle of a tri-
   angle is opposite the
   greater side.
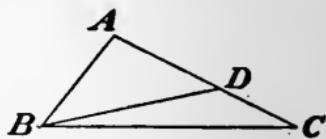
Let A C be greater than A B; take A D equal to A B,
and join B D.

Then since A D B is the exterior angle of the triangle
B D C, it is greater than the interior opposite angle
D C B.—Prop. 16.

But since the side A D is equal to the side A B, the angle
A D B is equal to the angle A B D.—Prop. 5.

Therefore the angle A B D also is greater than the angle
A C B.

Much more then is the angle A B C greater than the angle
A C B.—Ax. 9. Q. E. D.

IX.—FALLACIES

§ 133. <u>Any</u> <u>violation</u> of <u>logical law</u> is <u>a fallacy.</u>
Logical forms are determined originally by the
nature of intellect as expressed in the primary laws
of thought, from which are derived by deduction
the laws of special forms. Hence any essential de-
viation from a form is a violation of its law, and so
a fallacy. Under this wide definition come <u>illog-</u>
<u>ical</u> <u>predications,</u> <u>generalizations,</u> <u>definitions,</u> divi-
<u>sions,</u> etc., as well as <u>illogical inferences.</u>

Two remarks are needful. First, that <u>logical</u>
<u>forms,</u> though necessary, as stated in the definition
of logic (§ 1), <u>are nevertheless violable</u> (§ 5). They
are necessary to knowledge of truth, and cannot be
violated without risk of error, folly, falsity ; just
as a violation of the laws of health risks disorder,
disease, death. Second, that <u>what does not violate</u>
<u>logical law,</u> however false in matter, <u>is not fallacy.</u>
Our science does not take into consideration the
material truth or falsity of judgments (§§ 4, 50).
Therefore, in case of inference, the truth or falsity
of the premises and conclusion is disregarded, the
form alone being considered (§ 91). Many logi-
cians, overlooking this, include among fallacies syl-
logisms correct in form, but having false premises.

These, however, are not fallacies. For example, if some one argues from the distress of a country that the government is tyrannical, we must suppose him to assume that *Every distressed country is under tyranny*, which, though false, leads logically to his conclusion, and there is no fallacy; or that *Every country under a tyranny is distressed*, which may be true, but the inference from this, the middle being undistributed, is a *non sequitur*, a fallacy.

The distribution of fallacies is as follows:

Fallacies $\begin{cases} \text{Paralogisms} \\ \text{Sophisms} \quad \begin{cases} \text{In diction.} \\ \text{In matter.} \end{cases} \end{cases}$

The differences here indicated will be explained in the progress of the discussion.

§ 134. A paralogism is a violation of a law of form, manifest without regard to the diction or matter. Of this we have already had many incidental examples. When the form of a proposition is obviously the logical paradox, A=non-A, as *To do wrong is sometimes right;* or when there is an inference from All A is B, to All B is A, as *To possess a large amount of money is to be wealthy,* hence *To be wealthy is to possess a large amount of money;* or an inference through an undistributed middle; or an inference involving the illicit process—these and the like are paralogisms.

Sometimes, however, law is only apparently, not really, violated. For example:

> No rose is without thorns;
> This bouquet is of roses;
> ∴ This bouquet has thorns.

Here seems to be an affirmative conclusion from a negative premise, violating General Rule 4. But on looking into the diction of the major premise, it is seen to yield by infinitation *Every rose has thorns*, and then the form is Barbara.

§ 135. Sophisms in diction, *in voce*, are such as require an inspection of the expression in order to detect the formal fault. They all arise from ambiguities of language. A term repeated ambiguously, though identical to eye and ear, must be counted twice, for it represents two notions. A syllogism containing such a term is therefore, in thought, *quaternio terminorum*, a quaternion, a logical quadruped (§ 94). This is the common vice of sophisms in diction. Aristotle distinguishes six species, which we proceed to examine.

§ 136. **Æquivocatio** is the use of a term in two different senses. If it be the middle term, it is called the fallacy of ambiguous middle, as in:

> Designing persons are untrustworthy;
> Everybody forms designs;
> ∴ Nobody can be trusted.

Likewise an ambiguous major or minor term produces a quaternion.

Perhaps no fallacy is so prolific as this. Living languages abound in ambiguities, and no procedure

is safe that does not keep close watch upon them. Many important words, as *nature, state, representation, moral, inconceivable*, and even *money*, are quite ambiguous. There are at least five distinct senses in which the word *law* is habitually used. The only security is in exact definition and consistent usage. As by attrition crystals become pebbles, so words in common use lose their sharp meaning. Like coins defaced by much handling, current words are no longer clear. Science, to be accurate, takes refuge in a barbarous terminology.

The paranomasia or pun is the sophism of equivocation. Here is a time-honored example: *Two men ate oysters for a wager; one ate ninety-nine, but the other ate two more, for he ate a hundred and won.* This affords occasion for the general observation that jests are usually mock logic, and often in absurd form let fly a sharp dart of truth. The " bull " is a palpable self-contradiction, generally an unconscious blunder, but sometimes on purpose; as, *Do you believe in ghosts? No indeed, I've seen too many of them;* or, as when my wife said to me, *I hope I shall not live to see you a frisky widower.*

§ 137. **Amphibolia** differs from equivocation in that the ambiguity is in the construction of a sentence rather than in a term. Examples of amphiboly are: *How much is twice two and three? I will go and return to-morrow.* See Quince's prologue in *Midsummer-Night's Dream*, act v., sc. i. In the Nicene Creed, the words " by whom all

things were made" are grammatically referable either to the Father or to the Son. Amphiboly was the trick of the oracles. Thus the prophecy of the Spirit in *Henry VI.*, pt. ii., act i., sc. iv. :

> The duke yet lives that Henry shall depose,
> But him outlive, and die a violent death.

But this, says York, is just the response of the oracle to Pyrrhus :

> Aio te, Æacida, Romanos vincere posse;
> Ibis redibis nunquam in bello peribis.

§ 138. **Compositio et divisio** are conjoining what should be disjoined, and disjoining what should be conjoined. Thus, *He well knows the alphabet he had to learn ; In some things we offend all ; Moses was the daughter of Pharaoh's son ; Paul returned to his master one Simus (Onesimus).* Aristotle's example of composition is : *A man sitting can walk* (i. e., *can walk sitting*); of division, he gives : *5 is 2 and 3, both even and odd.* He treats them as distinct species, which seems unnecessary, since the distinction between them generally depends merely on which of the propositions, involved in the ambiguous statement, is granted, the affirmation of the other being the fallacy. Whately construes the above as *F. Compositionis*, thus :

> Two and three (*distributively*) are even and odd;
> Two and three (*collectively*) are five;
> ∴ Five is even and odd.

This is clear and correct, although it transposes the titles.

The distinction between fallacies of this class and amphibolia is not altogether clear. In many cases we hesitate. Perhaps to either may be referred ambiguities wrongly resolved by punctuation. A notable example is found in the United States Constitution, Art. 1, § 8. After the word "excises" a semicolon is frequently printed, whereas in the original draft, and in the authorized edition of March 3, 1877, it is followed by a comma. Alexander Hamilton held that the items of the rest of the section are additional powers; Madison, that they are limitations. The semicolon enlarges federal authority; the comma favors state-rights.

This gives occasion for the general observation that it should not be inferred from the trifling character of many of the examples used to illustrate fallacies, that the fallacies themselves are unimportant. In a brief trifle a point is often clearly exposed which, lurking in a body of weighty matters, may be fatal.

§ 139. **Accentus, prosodia,** resolves an ambiguity by a stress of voice so as to mislead, generally by an implication. The early rabbis laid emphasis on the word *neighbor* in *Thou shalt love thy neighbor* (Lev. xix. 18); hence their gloss, *and hate thine enemy* (corrected in Matt. v. 43 sq.). By emphasis on *against* in the ninth commandment, it is implied that one may bear false witness in favor of another, which was Jeanie Deans's temptation. The phrase *If you were' brave* differs from

If you were brave'. So also *Not the least difference* may mean no difference at all, or by varying the stress, a very considerable, perhaps the greatest difference. Some words, ambiguous to the eye, are resolved by accent, as *to con'jure*, to practise magic, and *to conjure'*, to entreat earnestly. Merely the tone may make all the difference between truth and falsehood.

Sarcasm is generally indicated by the circumflex accent, and unless this or certain tones are used, the meaning is perverted; as, *It cannot be that a prophet perish out of Jerusalem.* For other examples of divine irony, see 1 Kings xviii. 27; Job xii. 2; Psa. ii. 10; 2 Cor. xii. 13.

§ 140. **Figura dictionis** occurs when a metaphor or other figure of speech is construed literally, or *vice versa*, as:

> A fox is a quadruped;
> Herod is a fox;
> ∴ Herod is a quadruped.

This seems very trifling. But let it be observed that figurative expressions abound, that new matter can hardly be spoken of except metaphorically, that the history of the mental sciences shows how difficult it is to avoid being misled by material conceptions which are only remotely comparative, and that in debate illustrations are constantly mistaken for arguments, and often are more convincing than good logic. These considerations make it evident that this is a very subtle and ruinous form of fal-

lacy.   Hamilton speaks of it with great contempt, unaware that his famous argument for immediate perception is invalidated by this very sophism.

It is usual to include in this class errors arising from solecisms; as, *George Eliot deserves his fame.* Also those from paronyms; as, *Being touched with pity, his behavior was pitiful; A phenomenon is that which appears, and therefore is merely apparent.*

§ 141. Sophisms in matter, *in re*, are such as require an inspection of the matter in order to detect the formal fault.   They are quite commonly called "material fallacies," and described as those whose fault is not in form or diction, but in matter, meaning that the form is correct, but the matter, especially the premised matter, is false.   If so, they, being logically faultless, are not fallacies (§ 133).   But not so, for these sophisms are logically, formally faulty, only it is requisite to look beyond the diction and examine the matter in order to discover the fault.   Of this genus Aristotle distinguishes seven species, which we proceed to examine.

§ 142. **Accidens** arises from equating subject and accident, or whenever it is assumed that subject and accident have all their attributes in common.   By *accident* here is meant any subordinate part of a general notion, as in conversion *per accidens* (§ 82).   For example: *Men* (subject) *are bipeds; Birds are* (an accident of) *bipeds;* hence (equating subject and accident), *Men are birds,* or *Birds are*

men. But it is fallaciously assumed that *men* and *birds* have all other attributes in common. Obviously undistributed middle. Again : Since *Coriscus is not Socrates*, and *Socrates is a man*, it does not follow that *Coriscus is not a man*, because *Socrates*, who is denied of *Coriscus*, is merely an accident of *man*. Obviously illicit major. Again :

> The Greeks produced masterpieces of art;
> The Spartans were Greeks;
> ∴ The Spartans produced masterpieces of art.

Here *the Greeks*, the subject in the major premise, is the name of a genus taken as an undivided whole (§ 63), of which *the Spartans* is merely an accident. It is fallaciously assumed that whatever is attributable to the genus as such, may be attributed to an accidental member. Obviously ambiguous middle, and hence a quarternion.

§ 143. **Secundum quid** occurs in an inference *a dicto secundum quid ad dictum simpliciter*, and *vice versa*. It is the confusion of an absolute statement with one limited by time, manner, or some accidental relation.

The first infers from a statement made under an unessential restriction (*secundum quid*) to one made without restriction (*simpliciter*).

> Whatever is pernicious ought to be forbidden;
> The use of wine is pernicious;
> ∴ The use of wine ought to be forbidden.

Here the minor premise refers to wine used immoderately ; the conclusion, to wine however used.

This is the time-honored sophism of arguing against a thing from the abuse of it.

The second infers from a statement made without limitation to one limited, proceeding from what is essential to what is accidental.

> The meat you bought yesterday you ate to-day;
> You bought raw meat yesterday;
> ∴ You ate raw meat to-day.

Here is inferred, in the conclusion, of meat with the accidental quality of rawness added, what in the major is said of it simply, that is, of the essential substance, regardless of accidental qualities.

The first of these cases, when we look into the matter, may evidently be construed as illicit minor; for what is premised of some, a certain use of wine, is concluded of all use of wine. The second case is plainly a quaternion, having an ambiguous middle; for *The meat you bought yesterday* is used in two different senses — first, simply or essentially only; secondly, with its accident.

§ 144. **Ignoratio elenchi** is ignoring the refutation, answering to the wrong point, proving something not the contradictory (*elenchus*) of the thesis which one intends to overthrow. This supposes a disputant, an attempt at confutation. It is usual to take a wider view, and, under the title of Irrelevant Conclusion, or mistaking the issue, to include all cases where the attempt is to establish a thesis by a proof of something not sustaining it, or of something which may be mistaken for it. This

might well be termed *Ignoratio* or *Mutatio conclusionis.* Formally the fault is either in establishing something that is not the required contradictory of the thesis, or else establishing something that is not the required thesis.

§ 145. **Consequens** is to infer that the conclusion is false because a premise is false, or the argument unsound; also, to infer the truth of a premise from that of the conclusion. Thus, if some one argues for the existence of a God from its being universally believed, another may perhaps be able to refute the argument by producing an instance of a nation destitute of such belief, thus contradicting the minor premise; the argument ought then to go for nothing. But many think that this refutation disproves the existence of a God, in which they are guilty of illicit major; thus:

> Whatever is universally believed must be true;
> The existence of a God is not universally believed;
> ∴ The existence of a God is not true.

Others, again, from being already convinced of the truth of the first conclusion, the existence of a God, would infer the truth of the premise, which would be the fallacy of undistributed middle; thus:

> What is universally believed is true;
> The existence of a God is true;
> ∴ The existence of a God is universally believed.

If these two fallacies be put in hypothetical form, the one shall proceed from the denial of the antecedent to the denial of the consequent; the other

13

from affirming the consequent to the affirmation of the antecedent (§§ 91, 119). These two conditional fallacies, therefore, are thus identified respectively with those of illicit process and undistributed middle.

§ 146. **Petitio principii,** or petition, or begging the question, is the assumption, as a ground of proof, of a proposition that is not proved, or not granted, or not self-evident. It may occur in any one of five ways:

1st. When the question itself, the *quæsitum,* the very thing to be proved, is assumed. This may be concealed by using synonyms, or a name and its definition, either directly, or in a circumlocution. Thus there are two varieties.

Hysteron proteron, or the last first, does not extend beyond an epithet or a single proposition or inference. Thus: *rebel,* or *bigot.* Thus, synonymously: *The doctrine is heretical, for has it not, I beg, caused a schism in the Church?* Again:

A rectilinear figure of three sides has its angles equal to two right angles;

A triangle is a rectilinear figure of three sides;

∴ A triangle has its angles equal to two right angles.

Here the minor premise is a name and its definition. These being strictly identical notions (§ 35), differ, not in thought, but only in words; therefore the conclusion is assumed, or the question is begged by the major premise (§ 130). The formal fault of hysteron proteron, when syllogistic, is that there

are but two terms—a logical biped (§ 94). Typical forms are:

A is B		B is B
A is A	and	A is B
∴A is B		∴A is B

The conclusion is already in a premise, and nothing is proved. Cf. § 95, Ex. 12; and § 131 near end.

Diallelon, or a logical circle, occurs when a premise is repeated in a more remote conclusion. The form may be represented as a pro- and epi-syllogism, thus:

A is B		C is B
A is C	then	A is C
∴C is B		∴A is B

In this case the pro-syllogism has an illicit process, or else the epi-syllogism an undistributed middle. Of course any number of syllogisms, or a hiatus, may intervene, and more effectually conceal the fallacy. Plato, in the "Phædo," proves the immortality of the soul from its simplicity, and, in the "Republic," proves its simplicity from its immortality.

2d. When a particular is to be proved and a universal is assumed without warrant. Thus: *The king is tyrannical, for are not all kings more or less so?* This is not properly a fallacy, for the form is faultless (§ 133). Yet the major premise, being unproven, begs the question. It would be *petitio principii* to prove to a Mohammedan the divinity of Christ from New Testament texts, for he does not admit the authority of the Bible; but it would be a valid *argumentum ad hominem* (§ 108)

to prove to him from the Koran the prophetic mission of Jesus, for the authority of the Koran he acknowledges.

3d. When a universal is to be proved and a particular contained under it is assumed. Thus: *The knowledge of contraries is one, for is not the knowledge of black and white* (or *good and evil*, or any other pair of particular contraries) *one and the same?* This begs the question, but only in part. A deduction to all would be the illicit process (§ 79).

4th. When the problem to be proved is divided and its parts assumed in detail. Thus: *Medicine is the science of health and disease. For is it not the science of health? And also of disease?*

5th. When two facts are reciprocally implicated, and one is assumed to prove the other. Correlatives imply and are not inferred from each other (§ 78). Thus it is petition to say: *Alexander was the son of Philip, and therefore Philip was the father of Alexander; or, A spark caused the explosion, therefore the explosion was caused by a spark; or,—therefore the explosion was the effect of a spark.*

§ 147. **Non causa pro causa** assumes a premise which is not the cause to be the cause of an absurd conclusion. The conclusion may be a proper sequence, and its absurdity justify the contradiction of a premise, but not of the one assumed. Thus: *If the prisoner was one of the burglars, and made the foot-tracks under the window, then he was wear-*

Reasons are not Causes.

ing shoes half the size of his feet; but this is impossible; therefore he was not one of the burglars.
This *reductio ad absurdum* (§ 108) authorizes the denial of the second part of the protasis, but not of the first, with which the conclusion is not connected by any middle term, and so with the first part makes a quaternion. To detect the fallacy, examine whether the suppression of the contradicted premise would invalidate the sequence.

Evidently this sophism relates to *causa cognoscendi*, or reason only, not at all to *causa essendi* (§ 110). But treatises on logic quite commonly ignore the true sense, though the fallacy is by no means rare, and, misled by the *usus loquendi* of cause, say that it is "to mistake for a cause what is not a cause," meaning *causa essendi*. Thus: *A change of the moon causes a change in the weather; Cometa fulsit, ergo bellum erit.* This fallacy is the *Cum hoc, vel post hoc, ergo propter hoc.* It is an important fallacy of induction, but has no place in deduction.

§ 148. **Plures interrogationes** is the call for a single answer to plural questions asked in one. Thus: *Was Pisistratus the tyrant and scourge of Athens?* As he was the one but not the other, either a yea or a nay would commit the respondent to a false position. Avoiding one horn, he is caught on the other, and hence this sophism is sometimes called the Cornutus. A safe answer is, *Yes and no.* A variation in form is: *Are you the only rogue in*

your family? Such forms are much used in teasing, and lawyers badger unsophisticated witnesses in this way. To some compound question they demand what they call "a categorical answer," meaning a simple yea or nay, when either will entrap the witness in a damaging admission, or in a self-contradiction or other falsity. Again: *Why is a violincello player always fat?* But we should inquire *an sit?* before *cur sit?* The ancient example, *Have you cast your horns?* may be stated: *Either you have cast your horns, or you have them still; which?* But there is a *tertium* omitted: *or you have never had horns.* In this case it is the fallacy of incomplete disjunction (§ 114). All this seems quite frivolous, but it is not always so. Nor is the form necessarily fallacious. It is used by our Lord to entangle his adversaries (Matt. xxi. 24–27), in which case the disjunction is complete.

§ 149. **Praxis.** Designate and describe the paralogisms occurring in many, yet not in every one, of the following examples:

1. All plants come from seed, therefore all seeds come from plants.
2. The French Academy defined a crab as a small red fish that walks backwards. Very good, said Cuvier, only a crab is not a fish, is not red, and does not walk backwards.
3. A legitimate argument may fail to win assent;
   No fallacy is a legitimate argument;
   ∴ No fallacy can fail to win assent.

4. A mouse is an animal, therefore (by determination, § 80) a very large mouse is a very large animal.

5. Every one desires happiness; but virtue secures happiness; therefore every one desires virtue.

6. Only give me the luxuries of life, and I will dispense with the necessaries.

7. None but whites are civilized; the ancient Germans were whites; hence they were civilized.

8. None but whites are civilized; the East Indians are not whites; hence they are not civilized.

9. No good doctor ever takes fees; all good doctors are also lawyers; hence lawyers never take fees.

10. A little girl studying arithmetic, and coming to a "sum" in which oranges were exchanged for eggs, refused to try it, saying nobody would be such a fool.

11. J. S. Mill's introduction to his "Political Economy" is entitled "Preliminary Remarks," which proposes a prospective review.

12. A spaniel is defined as a species of the proximate genus dog.

13. Can you mention anything that is common property?

14. All that glitters is not gold; tinsel glitters; therefore tinsel is not gold.

15. Never do anything you need to be ashamed of, and then you need never be ashamed of anything you do.

16. All do not strive; but all wish to succeed; hence not all strive who wish to succeed.

17. Some possible cases are improbable;
∴ Some probable cases are impossible.

18. Liberty is a negation (absence of constraint);
    We cannot be conscious of a negative;
    ∴ We cannot be conscious of liberty.

19. Shakespeare knew little Latin and less Greek.

20. Touchstone says to Corin: Why, if thou never
    wast at court, thou never saw'st good manners;
    if thou never saw'st good manners, then thy
    manners must be wicked; and wickedness is sin,
    and sin is damnation. Thou art in a parlous
    state, Shepherd!

21. If some men are strong, it follows that some are weak.

22. An agnostic is one who holds that it is impossible
    to know anything with certainty.

23. There is no rule without exceptions;
    This statement is itself a rule;
    ∴ This statement has exceptions, or
    There are rules without exceptions.

24. If a wife be beautiful, she excites jealousy;
    If she be ugly, she excites disgust;
    Therefore it is best not to marry.

25. Whatever represses the liberties of mankind ought
    to be resisted; but among the things that do so,
    there are governments;
    ∴ Governments ought to be resisted.

26. Nothing is better than wisdom;
    Dry bread is better than nothing;
    ∴ Dry bread is better than wisdom.

27. Those to whom the Gospel promises come are the
    faithful;
    Many whom the world condemns are faithful;
    ∴ The Gospel promises come to many whom the
    world condemns.

Designate and describe the sophisms in diction occurring in many of the following examples:

28. Whoever necessarily goes or stays is not a free agent;
    But every one necessarily either goes or stays;
    ∴ No one is free.

29. Whatever a man walks on he tramples on;
    This man walks on the whole day;
    ∴ He tramples on the day.

30. The prophet spake to his sons, saying, Saddle me the ass; and they saddled *him*.

31. All criminal actions should be punished by law;
    Prosecutions for theft are criminal actions;
    ∴ Prosecutions for theft should be punished by law.

32. No moral principle is an animal impulse;
    But some animal impulses are principles of action;
    ∴ Some principles of action are not moral principles.

33. A member of Congress, charged with having called another a liar, apologized thus: It is quite true, and I am sorry for it.

34. Our consciousness testifies to the external reality of objects of sense-perception, but is its witness true? Of course, for your assertion, literally taken, means only this, that we are conscious of external reality.—*A reply to Hamilton.*

35. Thou shalt not bear false witness against thy *neighbor.*

36. The planets are seven; Mercury and Venus are planets;
    ∴ Mercury and Venus are seven.

37. Finis rei est illius perfectio;
    Mors est finis vitæ;
    ∴ Mors est vitæ perfectio.

38. Either animal or vegetable food may be altogether
     dispensed with ;
     All food is either animal or vegetable ;
     ∴ All food may be altogether dispensed with.
39. Philip saith to the eunuch, Γινώσκεις ἃ ἀναγινώσκεις ;
40. Pilate saith to the Jews, Behold your King !
     And they cried, Hail, King of the Jews !

Designate and describe the sophisms in matter
occurring in many of the following examples :

41. The gods, say the Epicureans, must be invested with
     the human form, because this form is most beau-
     tiful ; and everything beautiful must be found in
     them.
42. To pray for rain is to ask for a miracle ; but mira-
     cles have ceased. It is replied, first, that prayer
     for rain has often been followed by rain ; secondly,
     that men have succeeded in causing rain, and to
     say God cannot do what men can do is impious.
43. Prayer may be regarded as useful, if, indeed, we
     can regard our prayers as announcing to Deity
     what he does not know, or as effectual in chang-
     ing his purposes ;
     But we cannot tell the Omniscient what he does
     not already know, nor effect a change in his eter-
     nal purposes ;
     ∴ Prayer is useless.
44. The right of the government to command is un-
     questionable ; therefore we ought to obey it.
45. Unless logic profess to be an instrument of inven-
     tion, the reproach that it discovers nothing is
     unfounded ; but it does not make this profes-
     sion, and hence this reproach is unfounded.

46. Either God wills to remove evils and cannot; or
he can and will not; or he neither will nor can;
or he both will and can. If he will and cannot,
then he is weak, which is not true of God. If
he can and will not, then he is malicious, which
also is foreign to the nature of God. If he nei-
ther will nor can, then he is both malicious and
weak, and therefore cannot be God. If he both
can and will, which alone is consistent with the
nature of God, then whence are evils, or why
does he not remove them?

47. To allow every man freedom of speech must always
be, on the whole, for the good of the state; for
it is highly conducive to the interests of the
community that each individual should enjoy a
liberty, perfectly unlimited, of expressing his
sentiments on its affairs.

48. Mental effort promotes intellectual vigor, but wearies
the brain; hence what wearies strengthens; but
hard study is wearisome, and therefore strength-
ens the mind.

49. Whatever is true of John, Peter, etc., is true of all
mankind;
Mortality is true of John, Peter, etc.;
∴ Mortality is true of all mankind.

> —*Whately's "inductive syllogism," approved
> as such by Mill, Logic, bk. iii., ch. iii.*

50. This, that, and the other magnet attract iron;
This, that, and the other magnet represent all
magnets;
∴ All magnets attract iron.

> —*Hamilton's "inductive syllogism," Logic, § 62.*

51. What is not an uncommon occurrence may reason-
    ably be expected;
    To gain a high prize in a lottery is not an uncom-
    mon occurrence;
    ∴ To gain a high prize in a lottery may reasonably
    be expected.

52. He who calls you a man speaks truly;
    He who calls you a knave calls you a man;
    ∴ He who calls you a knave speaks truly.

53. Every effect is caused;
    The world is an effect;
    ∴ The world is caused.

54. Why does a ball, when dropped from the mast-head
    of a ship in full sail, fall not exactly at the foot
    of the mast, but nearer to the stern of the vessel?

55. Who is most hungry eats most;
    Who eats least is most hungry;
    ∴ Who eats least eats most.

56. Omne animal rationale est risibile;
    Omnis homo est animal rationale;
    ∴ Omnis homo est risibilis.

57. We are forbidden to kill;
    Inflicting capital punishment is killing;
    ∴ We are forbidden to inflict capital punishment.

58. He that is of God heareth the words of God: for
    this cause ye hear them not, because ye are not
    of God.—*John* viii. 47.

INDEX

(The number refers to the page.)

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